CARLOS HERNÁNDEZ MIRELES

Marketing Modeling for New Products

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Marketing Modellen Voor Nieuwe Producten

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Carlos Hernández Mireles
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Doctoral Committee

Promotor:	Prof.dr. P.H. Franses
Other members:	Prof.dr. B.J. Bronnenberg Prof.dr. P.K. Chintagunta Prof.dr. R. Paap
Co-promotor:	Dr. D. Fok

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To My Family
Carlos, Leticia, Tanhia and Ileana

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Carlos Hernández Mireles

Rotterdam, June 29th 2010

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Chapter 1

Introduction

Thesis Road-map

In this thesis we address the marketing of new products using mathematical and econometric models. We present a collection of models that are useful to study the following topics: (1) The optimal launch time of new and dominant technologies, (2) The triggers, speed and timing of new products' *price landings*, (3) The consumer heterogeneity that drives substitutions patterns. And, (4) the influential locations that drive the diffusion of new technologies.

These topics are explored in depth in the next four chapters and the topics of the chapters follow the order introduced above. Each chapter is self-contained and can be read independently from the others. However, the four chapters share a similar structure. That is, each chapter consists of an executive summary, a literature review, the modeling and econometric approach and its own conclusions or discussion.

The econometric approaches that we apply are diverse but they are mainly Bayesian. The exception is the second chapter where we apply non-linear least squares and simulation methods. The third chapter involves Bayesian mixture modeling. In the fourth chapter we present a new Bayesian approach for the random coefficient logit model. Finally, the study in the fifth chapter is based on Bayesian variable selection techniques and Bayesian spatial models.

In the next section we introduce the topics that we will explore in the next four chapters and we aim to give an impression and short overview of some of the important aspects related to the marketing of new products. The overview is based on Apple because the marketing techniques of this company offer a great setting related to the topics covered in this thesis. Note, however, that this thesis' research is not applied to Apple's products. After the overview, we conclude this introductory chapter with a summary of the academic contributions of this thesis.

The perfect marketing for new products?

When will Steve Jobs launch the next generation of the iPhone, the iPhone 4G? Hopefully for those working in marketing, Steve Jobs will prefer to launch the iPhone 4G at the time indicated by Apple's Vice-President (VP) of Marketing and at a time after the engineers and designers at Apple finished its technological development. But what will be the timing suggested by Apple's Marketing VP? Is it likely that the Marketing VP will strive to find the launch date that could result in the greatest consumer demand possible at all dates after the iPhone 4G launch? The question now seems to be when consumers, both current owners and non-owners of the iPhone, will purchase the iPhone 4G. Will they be anxiously waiting to purchase it as soon as it is available online or at their local Apple shop? Or will consumers wait some time after its introduction or will they even wait to leap-forward to a superior iPhone a couple of generations ahead, say, to the iPhone XG?

Currently, the iPhone is the leading and dominant technology in the smart-phone segment. One of the closest competitors of the iPhone is the BlackBerry produced by Research in Motion (RIM). How much do we know about the BlackBerry's "generations"? RIM managers decided to manage their products in a very complex generational series. Consumers have the option to buy the BlackBerry Bold 9700, the BlackBerry Storm2 9550, the Storm2 9530, the BlackBerry Curve 8900, the Curve 8500, the Curve 8300, the Bold 9000, the Tour 9630 and so on. Surprisingly, a similar generational marketing strategy is used by Nokia, Samsung and other phone manufacturers. That is, the current iPhone is competing against dozens of products. Is the communications market the only market where the iPhone is competing? The answer is no. The iPhone is the top ranking camera in Flickr and hence it may be the most popular device to make photos worldwide. The next most popular device in Flickr is the Canon EOS Digital Rebel, that is a Canon digital SLR![1] Moreover, the iPhone is becoming a popular gaming plat-

[1] See `http://www.flickr.com/cameras/` for the Flick rankings and `http://na.blackberry.com/eng/devices/` for the latest list of RIM devices.

form and it is competing also against the Nintendo DS and the PlayStation Portable. Each new market expands the market potential of the iPhone while at the same time each new market may be a call for tougher competition and retaliation. Later, we will refer to technologies that fight for dominance as *alpha technologies* because these markets resemble the struggle for dominance between, for example, *alpha* chimpanzees. The iPhone faces a market where it may be classified as the dominant and only *alpha* technology in the smart phone segment. However, a common setting consists of several *alpha* technologies all of which have the potential to become the market leader. That is the setting that we study in the second chapter. In the second chapter of this thesis we present a multi-generation model for new and dominant technologies. We specifically focus on the topic of the launch timing of *alpha* technologies and its optimality.

In all ways, Apple is doing a great effort to increase the desirability of its products much before their market launch and in fact, during all their life-cycles. If the marketing strategy is effective then the VP of Marketing could pick a launch date, for example, and then do her best to set an introductory price and launch Apple's product at a good timing relative to its marketing and advertising campaigns. The launch of the iPad has brought attention to Apple's pricing strategy. Not surprisingly, Apple aims to convince its consumers that the iPad is "a magical and revolutionary product at an unbelievable price". That is exactly the current main welcome message at `www.apple.com`. Of course, prices play an important marketing role and Apple has tried to manage the timing and depth of price cuts carefully. In general, prices of high-tech products show sudden transitions from initial high levels to permanent much lower levels. There may be many different reasons behind a price cut, like demand, competition, products release schedule or seasons, and Apple is adapting each of its products´pricing to their specific competitive and demand settings. Later, we will refer to these transitions as *price landings*. In the third chapter of this thesis we present an empirical study of *price landings* and their potential triggers. More specifically, we study the heterogeneity of *price landings* and our modeling approach uncovers the relative importance of different landing triggers.

The focus of Apple's marketing efforts varies per product. Recently, the advertising of Mac computers was focused on its product features, the technology. The "Hello, I'm a Mac" ads made special emphasis on the superiority of Mac computers relative to PC's. In contrast, the marketing for the iPhone was based on its applications ("Apps") while the Apps were not really an Apple's product. However, the flexibility, diversity and immense capabilities of these Apps was featured as the main product to advertise in the marketing campaign "There is an App for That". That is, the Marketing VP might have realized that network effects and the demand for software could increase the demand for the iPhone. The third example is the recent marketing campaign for the iPod and this time the focus were its users. The "dancing silhouettes" campaign featured only color silhouettes of iPod users dancing different types of music or it featured bands and their music, like U2 playing Vertigo. In summary, Apple is addressing consumer heterogeneity with brand-specific campaigns. In the fourth chapter we present a methodology that is useful to capture consumer heterogeneity and preference evolution based on aggregate sales data. Specifically, we present an approach that augments previous Bayesian analysis of the random coefficient logit model. We present a modeling approach that is new because it adds market-specific and global priors, time varying preferences and finally we model heterogeneity with a novel structure.

Overall, Apple's is known as a firm aiming to provide the best consumer experience and it is usually mentioned as a company with great customer service. There are, however, groups of customers that receive greater attention and these are Apple's fans. Steve Jobs manages and talks to this influential and selected group of consumers at different moments. The last time that Steve Jobs appeared on stage as key-note speaker was on January 27th of 2010 and he devoted a complete event to describe the features of the iPad to Apple fans and to the press. In addition, he announced the pricing for the iPad and its launch date. The iPad will be available at Apple stores on April 3rd 2010 and it can be pre-ordered since March 12th 2010. Influentials are people who have a significant effect on the behavior of others and they might be the engine of diffusion at different moments and locations. Hence, it is key to manage influentials and convince them about the marvels of products much before everyone else. Steve Jobs key-notes are always based in San Francisco but Apple fans are everywhere. Are these fans always influential? Do they play different roles during the life-cycle of new technologies? In the fifth chapter of this thesis we present an approach to find the influential locations that drive the diffusion of technologies in aggregate sales data and in location-specific online search data. We further provide insights on how the influential locations distribute in space and how they evolve in time.

Summary and Academic Contributions

The novelty of this thesis consists of the analysis of new or very recent data and the introduction of new marketing models.

The second chapter introduces a new diffusion model that is useful to analyze the optimal introduction timing of multi-generational technologies. Special focus is given to *firms' alpha*, that is the ability of a firm to transfer users of its old technologies to their new generations, and the effects of the *firms' alpha* on the introduction dates of potential dominant technologies. This same chapter's analyses are based on recent weekly data of game consoles and video-games and we provide new insights about the optimality of the launch timing of the Nintendo Wii and the PlayStation 3. Chapter 2 is joint work with Philip Hans Franses.

Next, in the third chapter, we present a new mathematical model for sudden price transitions. Surprisingly, we are the first to empirically model specifically these transitions, what we call *price landings*, and their triggers, timing and speed. Furthermore, our analysis is based in a new dataset containing almost 1200 recently introduced products. Our contribution offer insights into the heterogeneity of *price landings* and the untangling of the most likely triggers of *price landings* based on Bayesian mixture modeling. Chapter 3 is joint work with Dennis Fok and Philip Hans Franses.

The contribution in the fourth chapter is mainly the introduction of an augmented version of recent Bayesian analysis of the random coefficient logit model. The practical application of the Bayesian random coefficient model, specifically to large datasets, requires novel approaches and model formulations. We apply our new approach to both simulated data and to a unique and very large dataset of aggregate sales and our approach proves to be promising. Chapter 4 is joint work with Dennis Fok.

Finally, in the fifth chapter of the thesis we analyze new data collected from Google Insight and we apply recent Bayesian econometric approaches to identify influentials. We focus our analysis on the identification of the influential locations that drive the aggregate sales of new technologies. The specific techniques that we apply in this chapter, Bayesian variable selection and multivariate spatial models, are new to the marketing literature. Hence, our contribution consists of the illustration of how these techniques can be applied to study marketing problems while at the same time we provide insights about the time variation and spatial clustering of influentials.

Chapter 2

The Launch Timing of New and Dominant Multi-Generation Technologies

In this chaper we introduce a model that is suitable to study the diffusion of new and dominant multi-generation technologies. Examples are computer operating systems, mobile phone standards, video game consoles. Our model incorporates three main features. First, we add the ability of a firm to transfer users of its old technologies to the new generations, what we call *firms' alpha*. Second, we add competitive relations between market technologies. Third, the launch strategies diagnosed by our model cover, as special cases, the *now or never* strategies and hence it is suitable to study intermediate launch strategies.

We state the relationship of our model to previous research both in terms of the model formulation and in terms of some of its analytical solutions. Specifically, the model may reduce to the Bass or the Norton and Bass models. Regarding the analytical solutions, we find that the *launch never* strategy arises when there are late product introductions by competitors, when a *firm's alpha* is very low, or when the competition is intense while the *launch now* strategy arises only when a *firm's alpha* is zero.

In addition, we evaluate different launch strategies and the optimality of launch timings in two detailed case studies on the video game systems market. We study the portable systems (PS) and the video game consoles (VGC) industry. We present several insights from our analysis and we find interesting explanations for the pacing strategy in this market, for which we also provide a historical perspective.

We find that the appropriate timing of a new technology depends heavily on both the *firms' alphas* and on the competitive positioning of their products. In the VGC case we find that the Nintendo Wii was launched at an appropriate moment while the Sony PS3 perhaps should have never been launched.

2.1 Introduction

In a well-known study on the behavior of chimpanzees Jane Goodall writes:

> "In 1963 Goliath, a powerful and aggressive male in his prime (perhaps about 25 years of age) was the alpha male. He had a spectacular charging display during which he covered the ground very fast indeed, dragging and occasionally hurling branches. Early in 1964, however, Goliath was displaced from his top-ranking position in the community by an older and much less robust male, Mike... Unlike Goliath, who had maintained a very high ranking position for several years after losing his alpha rank, Mike dropped rapidly to a low position in the hierarchy... In chimpanzee society, dominance is something of a conundrum. The usual interpretation of the phenomenon is that it enables a high-ranking individual to have prior access to desirable foods, females, or resting places." (van Lawick-Goodall, 1973)

We believe that Goodall's description of *dominance* in the chimpanzee society directly applies to new technologies and their markets. Specifically, markets of new technologies formed by a few firms and products and by a single or a few dominant *alpha* technologies are analogous to the few chimpanzee males that fight for the *alpha* rank. Examples of products in this type of industries are operating systems, mobile phone standards, video game consoles, smart phones, and so on.

Many technology firms, like Apple or Microsoft, launch several versions of their products, what we know as product generations. Each time a new generation product is introduced to the market some or many of the users of the old generations switch to the new one, at the same time new users may adopt the new generation product while other users may switch from one firm´s products to another firm´s products after a new introduction. That is, each product generation cannibalizes its previous generation and each firm has a different capacity of transferring the users of the old technology to the new one. For example, we know that Apple has been very successful transferring the users of its old technologies to the new ones. Linux, even though it is a smaller player, is a second example of a technology with a high *alpha*. In contrast, it was widely documented how Microsoft users were hesitant to switch from Windows XP to Windows Vista. Some Windows users stickied to Windows XP while others switched to alternative operating systems. In this chapter we will refer to the firms capacity of cannibalizing and transferring users of old technologies to new ones as the *firm's alpha*. In our example, Apple would be the player with a high *alpha*.

In this chapter we extend the Norton and Bass (1987) model by incorporating three new elements that have not been addressed simultaneously in previous literature. These are the firm's ability of transferring its users to new technologies (the *firm's alpha*), the competitive interaction between firms in the market, and a new solution to the timing of new technologies. Our model is suitable to study the timing of new generation products in industries that are characterized by a relatively slow pace of introductions and a few firms launching new technologies. In addition, we test our model empirically under different settings and based on the new model we provide insights into the launch-timing strategies and into the optimality of launch timings.

Previous empirical literature has addressed the diffusion of new multi-generation technologies, like Norton and Bass (1987), Kim and Lee (2005), Danaher et al. (2001) and Kim et al. (2000), but they do not cover the topic of introduction timing. Two exceptions are Norton and Bass (1987) and Mahajan and Muller (1996). These last authors introduce the timing of new products into their models and tested them empirically. However, both the Norton and Bass (1987) and the Mahajan and Muller (1996) models suggest to launch new technology either *now or never*. Other analytical studies have addressed specifically the timing of new technologies, like Wilson and Norton (1989), Joshi et al. (2009), Bayus et al. (1997), Souza et al. (2004) and Morgan et al. (2001), but these later authors models have not been tested empirically and in most cases their models are suitable for industries with a fast pace of technology introductions, an exception being Joshi et al. (2009). More importantly, these studies do not incorporate the three new elements we address simultaneously.

The plan of the chapter is as follows. In Section 2.2 we present our literature review. In Section 2.3 we present our model for the duopoly and triopoly case (sections 2.3.1 and 2.3.2, respectively), we discuss its relationship to previous models (section 2.3.3) and the analytical properties that distinguish it from previous models (section 2.3.4). In Section 2.4 we introduce the market context and our data. In Section 2.5 we motivate the model assumptions and the estimation procedure. In Section 3.5 we discuss the estimation results. In the next two sections we use our model to study the industry. In Section 2.7 we study the portable system market and we give insights about different launch strategies. Next, in Section 2.8, we study the main video game console market, composed of Microsoft, Sony and Nintendo, and we focus our analysis in the latest console race. We further provide insights into how different introduction timings may be optimal. Finally, in Section 3.6 we present our discussion and conclusions.

2.2 Literature Review

To our knowledge, Wilson and Norton (1989) and Mahajan and Muller (1996) are the two key studies concerned with the question of when it is optimal for a monopoly to launch multi-generation products. According to Wilson and Norton (1989) there are three critical issues which affect the optimal introduction time of a new generation. These are the interrelationship of sales of the two products, their profit margins and the planning horizon. Surprisingly, their model provides two optimal solutions regardless of the relevance of these factors. They conclude that different generations of a product should be introduced either all at the same time or sequentially and not overlapping. In a similar vein, Mahajan and Muller (1996) conclude that a new generation should be introduced as soon as it is available (if its market potential is larger than the preceding one) or it should be delayed to a much later stage, that is, to the maturity of the previous generation. Their findings seem special cases of the solutions proposed by Kamien and Schwartz (1972). Kamien and Schwartz (1972) suggest to *never launch* a technology only under extreme competition and to *launch now* only if the firm needs to take advantage of a profit stream that would otherwise be smaller once competitors come in.

More recently, Joshi et al. (2009) study the problem of product launch timings across different markets. They characterize situations, depending on social influence, where it is optimal to launch before maturity or after the maturity of the first generation product. However, Joshi et al. (2009) do not incorporate competition and their model is only useful to study the interaction of products across markets (same product in two geographies, for example). Souza et al. (2004) study the new product introduction strategy and its relation to industry clock speed. They provide analytical evidence that a time-pacing strategy (launching products every n time periods) performs relatively well compared to the optimal strategy. Their model applies to settings with a high frequency of product introductions. The studies of Morgan et al. (2001) and Bayus et al. (1997) analyze how the trade-offs between quality or product performance (measured by development costs) interact with the introduction timing decision. In contrast, we study the relationship between cannibalization and competition with the introduction timing decisions.

The literature on multi-generation products is very extensive. Padmanabhan and Bass (1993a) and Bayus (1992) propose models to price successive generations of products, Danaher et al. (2001) analyze the relation between the marketing mix and diffusion of multi-generation products, Bucklin and Sengupta (1993) examine the diffusion of complementary innovations, Kim et al. (2001), Chatterjee and Eliashberg (1990), Kim and Srinivasan (2001), Jun and Park (1999), Vakratsas and Bass (2002) and Bayus (1991) study how and when consumers decide to upgrade to improved products' versions. Islam and Meade (2000), Islam and Meade (1997) and Olson and Joi (1985) propose models for diffusion and replacement of products, while Purohit (1994), Robertson et al. (1995) and Prasad et al. (2004) analyze the introduction strategies of multi-generations products or the release of single products in multiple channels. Finally, Kim et al. (2000), Kim and Lee (2005), Peterson and Mahajan (1978) and Islam and Meade (1997) present alternative diffusion models for successive generations of products.

Our contributions to this literature are as follows. First, we propose a model that incorporates competition and cannibalization (*firm's alpha*) based on a duopolistic and triopolistic market. Second, our model parameters are simple to estimate or to calibrate with secondary quantitative or qualitative information and it is possible to find intermediate solutions to the introduction timing problem. Third, we provide two detailed case studies about the timing of game systems that are not documented in the literature. Finally, we present new insights regarding different launch strategies and the optimality of timing decisions.

Next we briefly discuss the Norton and Bass Model (NBM) as it is our departing point and it is essential in our model development.

2.2.1 The Norton and Bass Model

In this chapter we overcome three limitations of the NBM model that have not been jointly addressed in previous research. Denote $S_1(\tau_1, \tau_2)$ as the first generation sales, $S_2(\tau_1, \tau_2)$ as the second generation sales and denote τ_1 and τ_2 as the launch moment of these generations, respectively. The first limitation is that $\partial(S_1(\tau_1, \tau_2) + S_2(\tau_1, \tau_2))/\partial\tau_2 = 0$ is obtained when $\tau_2 = 0$ or when $\tau_2 = \infty$. $S_g(\tau_1, \tau_2)$ are the sales of generation g given the introduction timings of the first and second generation products, τ_1 and τ_2, respectively. Therefore, the basic Norton and Bass (1987) model is not helpful to derive an intermediate optimal introduction timing apart of these two solutions. The second limitation is that it assumes that all the sales of the previous generation are captured by the second generation. Finally, the NBM does not consider the diffusion of competing products.

In the NBM cumulative sales are proportional to the cumulative distribution function of the adoption rate $F(t)$ and the market potential m. When a second generation is introduced, substitution and adoption effects should be added to the previous equation. For the case of two generations, Norton and Bass posit that the first generation cumulative sales follow

$$S_1(\tau_1, \tau_2) = m_1 F_1(\tau_1)[1 - F_2(\tau_2)], \text{ for } t > 0, \tag{2.1}$$

and that the second generation follows

$$S_2(\tau_1, \tau_2) = F_2(\tau_2)[m_2 + F_1(\tau_1)m_1], \text{ for } t > \tau_2 \tag{2.2}$$

where we use $S_g(\tau_1, \tau_2)$ to refer to the vector $[S_g(\tau_1, \tau_2; t = 0), \ldots, S_g(\tau_1, \tau_2; t = T_p)]$ and $S_1(\tau_1, \tau_2; t)$ is equal to $m_1 F_1(\tau_1; t)[1 - F_2(\tau_2; t)]$ while $S_2(\tau_1, \tau_2; t)$ is equal to $F_2(\tau_2; t)[m_2 + F_1(\tau_1; t)m_1]$. The introduction date of the first generation ($g = 1$) is τ_1 and the introduction date of the second generation ($g = 2$) is τ_2. T_p is the planning horizon, which is set as ∞ in Norton and Bass (1987). $F_i(\tau_i; t)$ is the cumulative sales function of generation g defined as $F_g(\tau_g; t) = [1 - e^{-b_i(t-\tau_g)}/1 + a_i e^{-b_g(t-\tau_g)}]$ for $t > \tau_g$ and a_g=q_g/p_g and $b_g = p_g + q_g$, $g = 1, 2$. We use $F_g(\tau_g)$ to refer to the vector $[F_g(\tau_g; t = 0), \ldots, F_g(\tau_g; t = Tp)]$. Slightly stricter notation would use $F_g(\tau_g; t, \theta)$ where $\theta = (p_g, q_g, m_g)$ but we use the former as we focus on the timing parameters in this study. Note that in the Norton and Bass (1987) τ_1 is assumed to be fixed at some value (possibly at $t = 0$) and they do not focus on its value.

The equations of the NBM posit that after the second generation is introduced at time τ_2, the first generation's cumulative sales $S_1(\tau_1, \tau_2)$ become proportional to its cumulative adoption function $F_1(\tau_1)$, its market potential m_1, and the sales not captured by the second generation $[1 - F_2(\tau_2)]$ after τ_2. The sales of the second generation $S_2(\tau_1, \tau_2)$ are proportional to their own market potential m_2 and to the cumulative sales of the first generation $F_1(\tau_1)m_1$ after τ_2.

If the NBM equation (2.1) would contain only the term $m_1F_1(\tau_1)$, then the sales $S_1(\tau_1, \tau_2)$ will be equivalent to the model of Bass (1969). However, in the Norton and Bass (1987) model a fraction $F_2(\tau_2)$ of $m_1F_1(\tau_1)$ is captured by the second generation. Consequently, there is a moment in time when $F_2(\tau_2)$ will become 1 and all of the first generation sales are transferred to the second generation and the last element of $S_1(\tau_1, \tau_2)$ becomes 0. At the same time $S_2(\tau_1, \tau_2) = m_2F_2(\tau_2) + F_2(\tau_2)F_1(\tau_1)m_1$ and therefore, $m_1 + m_2$ is the last element of the vector $S_2(\tau_1, \tau_2)$, given in equation (2.2).

In the next section we present a model that is a generalized version of the NBM and we believe this new general model overcomes all the three limitations of the NBM.

2.3 A Multi-Product Diffusion Model with Competition

This section is divided in four subsections. In the first (subsection 2.3.1) we extend the NBM to the duopoly case and in the second (subsection 2.3.2) we extend the model to the triopoly case. Both extensions are based on the same assumptions and we present the duopoly case first for ease of exposition. In the third section we present the relationship of our model to previous models proposed in the literature (section 2.3.3). Finally, in the fourth (subsection 2.3.4) we present the intuition and the analytical properties that make our specification suitable to optimize and study the launch timing of new dominant technologies.

2.3.1 Duopoly Multi-Generation Model

In order to expand the Norton and Bass (1987) model and add a second firm or a second competing product, we should make assumptions about the relationship between the firms' products. Here we make the assumption that the relationship between the two generations products of a firm are related in a very similar but more flexible way than in the NBM, and that is where the *alpha* parameter comes in. Additionally, we will assume that the sales that go from one product to a competitor's version are proportional to the cumulative sales function of the competitor's products.

Formally, if the market is composed of *two* firms s and n, the cumulative sales of firm s are

$$S_1^s(\tau_1^s, \tau_2^s | \tau_1^n, \tau_2^n) = \tilde{S}_1^s(\tau_1^s, \tau_2^s)[1 - \phi_{11}^{sn} F_1^n(\tau_1^n)][1 - \phi_{12}^{sn} F_2^n(\tau_2^n)] \tag{2.3}$$

and

$$S_2^s(\tau_1^s, \tau_2^s | \tau_1^n, \tau_2^n) = \tilde{S}_2^s(\tau_1^s, \tau_2^s)[1 - \phi_{21}^{sn} F_1^n(\tau_1^n)][1 - \phi_{22}^{sn} F_2^n(\tau_2^n)] \tag{2.4}$$

The cumulative sales of firm n are

$$S_1^n(\tau_1^n, \tau_2^n | \tau_1^s, \tau_2^s) = \tilde{S}_1^n(\tau_1^n, \tau_2^n)[1 + \phi_{11}^{ns} F_1^s(\tau_1^s)][1 + \phi_{12}^{ns} F_2^s(\tau_2^s)] \tag{2.5}$$

and

$$S_2^n(\tau_1^n, \tau_2^n | \tau_1^s, \tau_2^s) = \tilde{S}_2^n(\tau_1^n, \tau_2^n)[1 + \phi_{21}^{ns} F_1^s(\tau_1^s)][1 + \phi_{22}^{ns} F_2^s(\tau_2^s)] \tag{2.6}$$

where $\tilde{S_1^i}$ and $\tilde{S_2^i}$ are defined as

$$\tilde{S_1^i}(\tau_1^i, \tau_2^i) = m_1^j F_1^i(\tau_1^i)[1 - \alpha_i F_2^i(\tau_2^i)] \text{ for } i = n \text{ or } s \tag{2.7}$$

and

$$\tilde{S_2^j}(\tau_1^i, \tau_2^i) = F_2^i(\tau_2^i)[m_2^i + \alpha_i F_1^i(\tau_1^i) m_1^i] \text{ for } i = n \text{ or } s \tag{2.8}$$

Finally we have that

$$F_g^i(\tau_g^i; t) = [1 - e^{-b_g^i(t-\tau_g^i)} / 1 + a_g^i e^{-b_g^i(t-\tau_g^i)}] \times I(\tau_g^i \geq t) \text{ for } t > 0 \tag{2.9}$$

where $S_g^i(\tau_1^i, \tau_2^i | \tau_1^s, \tau_2^s)$ represent the sales of generation g of firm i achieved by launching its first and second generation products at τ_1^i and τ_2^i and given the competing firm s launched its products at τ_1^s and τ_2^s; $a_g^i = q_g^i/p_g^i$ and $b_g^i = p_g^i + q_g^i$ and $I(\tau_g^i > t)$ is an indicator function that equals 1 when the introduction time of generation g of firm i, τ_g^i, is larger than or equal to t and zero otherwise. The term ϕ_{gk}^{ij} refers to the substitution (or loyalty) parameter between the generation g of firm i and the generation k of firm j. We use $F_g^i(\tau_g^i)$ to represent the vector $[F_g^i(\tau_g^i; t = 0), \ldots, F_g^i(\tau_g^i; t = T_p)]$. Again, stricter notation would use $F_g^i(\tau_g^i; t, \theta)$ where θ is a vector that collects all other parameters in the model. The parameters p_g^i and q_g^i are the innovation and imitation parameters of generation g and firm i, respectively, $g = 1, 2$ and $i = n, s$.

We may refer occasionally to ϕ as the vector $(\phi_1, \ldots, \phi_N)$ where N is the number of products and to α as the vector $(\alpha_1, \ldots, \alpha_I)$ where I is the number of firms. Equations 2.3 to 2.9 allow for a wide variety of relationships given the sign and size of what we call the loyalty parameters or ϕ and the values of the the *alpha* cannibalization parameters (α). The role of the α parameter is to relax the assumption of the NBM that all the sales of the first generation of a firm are transferred to the second generation. Note that the last elements of the vector in $\tilde{S}_2^j(\tau_1^i, \tau_2^i)$ will be equal to $m_2^i + \alpha m_1^i$ and the last element of $\tilde{S}_1^i(\tau_1^i, \tau_2^i)$ is equal to $m_1^j - \alpha m_1^j$. Therefore α can be interpreted as the proportion of sales that the first generation transfers to the next when $t = T_p$ and T_p is of course sufficiently long.

In Figure 2.1 we sketch the relationship between product generations in the duopoly model. Basically, there is substitution between all products but substitution starts at different points in time. The first generation is launched at $t = 0$ and it is the only product in the market up to $t = T1$. At this moment the first generation of the second firm is launched and the substitution between these two products (represented by the blank continuous line) starts too. The rest of the products are launched at time $t = T2$ and $t = T3$ and the substitution between them and the products launched before them start at these times. Note that the model allows for the possibility of *never* launching a product if we set its launch date at $t = T_p$. This figure represents a hypothetical case of launch dates but we can evaluate any launch-timing in the model. For example, we could evaluate the result of launching the products in reverse order or in any order. In practice the second generation arrives after the first one, but any other combination is allowed. Finally, note that there is only one single arrow between the products in the figure. That is, we assume symmetric competitive parameters. If the relationship between products is not symmetric then we would need two arrows connecting any pair of products in Figure 2.1.

Next we present the triopoly model and at the end of next section we discuss how both the duopoly and the triopoly models are related to previous research.

2.3.2 Triopoly Multi-Generation Model

In this section we extend the duopoly model and set the sales equations for firms s, n and x and we hold the assumption that each firm sells two generations of the same product.

The cumulative sales equations for firm x are:

$$\begin{aligned} S_1^x(\tau_1^x,\tau_2^x|\tau_1^s,\tau_2^s,\tau_1^n,\tau_2^n) &= \tilde{S}_1^x(\tau_1^x,\tau_2^x)[1+\phi_{11}^{xs}F_1^s(\tau_1^s)] \\ &\times [1+\phi_{12}^{xs}F_2^s(\tau_2^s)][1+\phi_{11}^{xn}F_1^n(\tau_1^n)][1+\phi_{12}^{xn}F_2^n(\tau_2^n)] \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} S_2^x(\tau_1^x,\tau_2^x|\tau_1^s,\tau_2^s,\tau_1^n,\tau_2^n) &= \tilde{S}_2^x(\tau_1^x,\tau_2^x)[1+\phi_{21}^{xs}F_1^s(\tau_1^s)] \\ &\times [1+\phi_{22}^{xs}F_2^s(\tau_2^s)][1+\phi_{21}^{xn}F_1^n(\tau_1^n)][1+\phi_{22}^{xn}F_2^n(\tau_2^n)] \end{aligned} \tag{2.11}$$

The cumulative sales equations for firm s are:

$$\begin{aligned} S_1^s(\tau_1^s,\tau_2^s|\tau_1^x,\tau_2^x,\tau_1^n,\tau_2^n) &= \tilde{S}_1^s(\tau_1^s,\tau_2^s)[1-\phi_{11}^{sx}F_1^x(\tau_1^x)] \\ &\times [1-\phi_{12}^{sx}F_2^x(\tau_2^x)][1+\phi_{11}^{sn}F_1^n(\tau_1^n)][1+\phi_{12}^{sn}F_2^n(\tau_2^n)] \end{aligned} \tag{2.12}$$

and

$$S_2^s(\tau_1^s, \tau_2^s | \tau_1^x, \tau_2^x, \tau_1^n, \tau_2^n) = \tilde{S}_2^s(\tau_1^s, \tau_2^s)[1 - \phi_{21}^{sx} F_1^x(\tau_1^x)] \\ \times [1 - \phi_{22}^{sx} F_2^x(\tau_2^x)][1 + \phi_{21}^{sn} F_1^n(\tau_1^n)][1 + \phi_{22}^{sn} F_2^n(\tau_2^n)] \quad (2.13)$$

And, the cumulative sales equations for firm n are:

$$S_1^n(\tau_1^n, \tau_2^n | \tau_1^x, \tau_2^x, \tau_1^s, \tau_2^s) = \tilde{S}_1^n(\tau_1^n, \tau_2^n)[1 - \phi_{11}^{nx} F_1^x(t - \tau_1^x)] \\ \times [1 - \phi_{12}^{nx} F_2^x(\tau_2^x)][1 - \phi_{11}^{ns} F_1^s(\tau_1^s)][1 - \phi_{12}^{ns} F_2^s(t - \tau_2^s)] \quad (2.14)$$

and

$$S_2^n(\tau_1^n, \tau_2^n | \tau_1^x, \tau_2^x, \tau_1^s, \tau_2^s) = \tilde{S}_2^n(\tau_1^n, \tau_2^n)[1 - \phi_{21}^{nx} F_1^x(\tau_1^x)] \\ \times [1 - \phi_{22}^{nx} F_2^x(\tau_2^x)][1 - \phi_{21}^{ns} F_1^s(t - \tau_1^s)][1 - \phi_{22}^{ns} F_2^s(t - \tau_2^s)] \quad (2.15)$$

where $\tilde{S}_1^i$ and $\tilde{S}_2^i$ are defined as

$$\tilde{S}_1^i(\tau_1^i, \tau_2^i) = m_1^i F_1^i(\tau_1^i)[1 - \alpha_i F_2^i(\tau_2^i)] \text{ for } i = n \text{ or } s \text{ or } x \quad (2.16)$$

and

$$\tilde{S}_2^i((\tau_1^i, \tau_2^i) = F_2^i(\tau_2^i)[m_2^i + \alpha_i F_1^i(t - \tau_1^i) m_1^i] \text{ for } j = n \text{ or } s \text{ or } x \quad (2.17)$$

and

$$F_g^i(\tau_g^i) = [1 - e^{-b_g^i(t - \tau_g^i)} / 1 + a_g^i e^{-b_g^i(t - \tau_g^i)}] \times I(\tau_g^i \geq t) \text{ for } t > 0 \quad (2.18)$$

where $a_g^i = q_g^i / p_g^i$ and $b_g^i = p_g^i + q_g^i$ and $I(\tau_g^i > t)$ is an indicator function that equals 1 when the introduction time of generation g of firm i, τ_g^i, is larger than or equal to t and zero otherwise. The term ϕ_{gk}^{ij} is the competitive parameter that relates the generation g of firm i with the generation k of firm j. The parameters p_g^i and q_g^i are the innovation and imitation parameters of generation g, respectively, $g = 1, 2$.

The specification of (2.10) to (2.18) is similar to the duopoly case but now we allow for substitution between three market players x, s, and n and each of their products. The duopoly model consists of four launch-timing parameters, eight ϕ parameters, two α parameters, four p and q parameters and four m parameters. That is in total 26 parameters in four equations. The triopoly model consists of 45 parameters (six τ, 24 ϕ, six p and q, three α and six m) in six equations. In the estimation section 2.5 we describe how we calibrate both models and the parameter restrictions and assumptions we use. Next we describe the relationship of our model with previous models.

2.3.3 Links with Other Models

In Figure 2.2 we summarize the relationship of this general NBM with previous models based on different parameter configurations. It is useful to see the nodes at the top of the figure as possible cases for each firm in our model. We start with the left node. If the α parameter, in one of the firm's equations, is equal to zero then there exists no cannibalization between a specific firm generations and the diffusion of each of its generations follows an independent Bass Model. However, in this case if some of the ϕ parameters are different from zero then we have independent Bass Models but we add inter-generation competition (or what is the same as between firms competition); otherwise they follow independent Bass models. On the right hand side of the figure we see the case when the α parameter is set to 1 and this means that the relationship of generations within firms follows the NBM specification. As in the previous node the ϕ parameters may add inter-generation competition between firms (note that is not within the same firm). Finally, in the central node we have the case when α is different from both 0 and 1. In this last case, the model allows cannibalization within a firm's generations but the cannibalization is different from the NBM. Therefore we call this a *second type of cannibalization.* As before, for this node the ϕ parameters may add inter-generation competition between firms.

At the bottom of Figure 2.2 we give three boxes representing firms and the arrows correspond to two hypothetical specifications (case 1 and 2) for each firm. In the first case, firm 1 products follow a NBM with *second type of cannibalization*, firm 2 products follow independent Bass Models while firm 3 products follow the NBM. That is, in this case the only firm facing the effects of competition is firm 2. In the second case we set a different combination and our intention is to illustrate that the model parameters allow a diverse set of diffusion patters among firms and products. A similar specification for the NBM is possible when either the p or q of any of the generations is equal to 0. Note that each firm launches two generations of products within the planning horizon but the triopoly model may reduce to the duopoly model in case a firm sets the launch date at the end of the planning horizon (what we refer as T_p) for its two generations. A different specification happens when each firm launches a single product by setting one of its generations launch-timing equal to T_p. Hence, our model is flexible enough to allow different substitution patterns between firms' products and within firm generations. At the same time the triopoly case might reduce to different number of firms or products depending on the parameter values.

2.3.4 Why Our Model Works

In this subsection we present the intuition of why our model is useful to find intermediates dates rather than $\tau = 0$ or $\tau = \infty$ solutions of the NBM. The intermediate solutions are possible due to the trade-off between competitive interaction between products and the cannibalization within a firm's generations. For example, if the firm n launches a product at time τ_c and this product might enhance/deter the sales of one of the products of firm s after this time. Then the firm s has the incentive to advance/postpone the launch of its product relative to the launch of the competing product. In this way, firm s could maximize/minimize the positive/negative effects of competition. That is, the timing decision depends on the sign and size of the effect of firm's n product on the sales of firm's s products. In addition, there is a trade-off between maximizing or minimizing the effect of competition and the effects on firm s previous generation product. Therefore, by launching the second generation sooner the previous generation might lose sales to the second generation earlier in time. In summary, the optimization of the competitive effects and the own cannibalization effects is possible in our specification while it is not possible to optimize them in the NBM.

Here we present a simplified version of the duopoly model and assume that one of the competing firms launches only one product at τ_c while the second firm s sells two products and these are launched at τ_1 and τ_2. We further assume that the competitive effects are measured by the coefficients ϕ_1 and ϕ_2. Formally, the equations of firm s are

$$S_1^s(\tau_1, \tau_2|\tau_c) = m_1 F_1^s(\tau_1)[1 - \alpha_s F_2^s(\tau_2)][1 - \phi_1 F_1^c(\tau_c)], \text{ for } t > 0, \tag{2.19}$$

and

$$S_2^s(\tau_1, \tau_2|\tau_c) = F_2^s(\tau_2)[m_2 + \alpha_s F_1^s(\tau_1) m_1][1 - \phi_2 F_1^c(\tau_c)], \text{ for } t > \tau_2 \tag{2.20}$$

That is, the first and second generation sales of firm s, $S_1^s(\tau_1, \tau_2|\tau_c)$ and $S_2^s(\tau_1, \tau_2|\tau_c)$, are now related to the competing product by the loyalty parameters ϕ_1 and ϕ_2. It is easy to show that the sales gained or lost by adding competition to the NBM (with cannibalization of type 2) are

$$\Delta_s = \big[\alpha_s(\phi_2 - \phi_1) m_1 F_1^s(\tau_1) F_2^s(\tau_2) + \\ \phi_1 m_1 F_1^s(\tau_1) + \phi_2 m_2 F_2^s(\tau_2)\big] F_1^c(\tau_c), \text{ for } t \geq \tau_c \tag{2.21}$$

Δ_s is the sales change due to the introduction of a competing product and it depends on the parameters α_s, ϕ_1 and ϕ_2 and on the introduction timings τ_1 and τ_2 relative to τ_c. The terms $\phi_1 m_1 F_1^s(\tau_1)$ and $\phi_2 m_2 F_2^s(\tau_2)$ measure the share of each product of firm s that might be transferred/received to/from a competing product and the shares are ϕ_1 and ϕ_2. The term $\alpha_s(\phi_2 - \phi_1) m_1 F_1^s(\tau_1) F_2^s(\tau_2)$ reflects the share of the cannibalized sales that might be transferred to a competing product and this share is $\alpha_s \times (\phi_2 - \phi_1)$. Note that α_s is the share transferred between generations of the firm s while $\alpha_s \times (\phi_2 - \phi_1)$ is the share that might be transfer to a competing product. If $\alpha_s = 0$ this implies no cannibalization and we are back to the NBM specification with competition. Finally, all terms belonging to firm s interact with the diffusion of the competing product $F_1^c(\tau_c)$ after τ_c. This last term exists only after $t > \tau_c$ and hence firm s decision should take into account that after time τ_c their products will gain or lose some share to the competing product. Note that equation (2.21) uses a simplified version of the duopoly model and that in our application below we use the complete duopoly and triopoly model.

The following lemmas cover a few interesting optimal timing scenarios. We include them because they illustrate some extreme cases where the *launch now or never* strategy may be valid and they illustrate the flexibility of our model specification.

Lemma 1 *The optimal introduction timing of both the first and second generation products is equal to zero when there is no cannibalization ($\alpha_s = 0$), when the $\phi_1 < 0$ and $\phi_2 < 0$ and there is one competitive introduction at τ_c.*

From (2.21) it follows that if $\alpha_s = 0$, one has $\Delta_s = -(\phi_1 m_1 F_1^s(\tau_1) + \phi_2 m_2 F_2^s(\tau_2))F_1^c(\tau_c)$. It is clear that both products should be introduced at $t = 0$ given that they face competition after τ_c, that is, the earlier they are both introduced, the better. Hence, in the case of no cannibalization with competition the option of *launch now* is the optimal solution. If there is no competition and cannibalization we are back to the solutions of the Norton and Bass model. This lemma is in line with Kamien and Schwartz (1972).

Lemma 2 *The optimal introduction timing of the first and second generation products (τ_1 and τ_2) are equal to τ_c when there is no cannibalization ($\alpha_s = 0$), when the $\phi_1 > 0$ or the $\phi_2 > 0$, respectively, and when a competitive introduction happens at τ_c.*

Introducing at time τ_c produces a positive Δ_s and it is clear that a firm should choose a time closer to τ_c. If both products are launched before τ_c the sales stream is smaller between τ_1 and τ_c for the first generation, and they are smaller between time τ_2 and τ_c for the second generation. On the other hand, if they are launched after τ_c they do not benefit from competition for $\tau_1 - \tau_c$ or $\tau_2 - \tau_c$ periods, respectively. This lemma implies that *imitation* may be optimal under certain conditions. As before, the strategy of *launch never* is discarded because there are positive returns to launch at dates closer to competitors. This lemma may be modified easily to the situation where imitation is optimal for only one generation, for example if $\phi_1 = 0$ and $\phi_2 > 0$. In our application below we will conduct a numerical exercise (in section 2.7.2) where this lemma is at work.

Lemma 3 *It is optimal to never launch the second generation when $S_2^s(\tau_1, \tau_2|\tau_c) + \Delta_s < 0$.*

When the returns on introducing the new product Δ_s outweigh the unit sales of $S_2^s(\tau_1, \tau_2|\tau_c)$ then it is optimal not to introduce it. Hence, the *launch never* strategy arises when there is stiff competition as in Kamien and Schwartz (1972). In our case study (section 2.8.2) we evaluate the parameter space that leads to this lemma.

There are other interesting possibilities of intermediate launch-timings when there is cannibalization and competition either for the first or second generation given different values for the ϕ_1, ϕ_2 and α parameters. In our case studies we explore numerically other possibilities for the α parameter and the optimal timing of products and explore the parameter space that may lead to any of these lemmas or to the *launch now or never* strategy.

2.4 The Video Game Hardware Market

The hardware market for video games can be split in two sub-markets: hardware for portable systems (PS) and hardware for video game consoles (VGC). In this chapter we treat these markets to be independent of each other. Indeed, most press articles indicate that the markets of PS and VGC are independent. See for example The Herald (2005), Financial Times (2004), The Economist (2004) and The Washington Post (2008). The reader may be familiar with the video game console wars between Microsoft, Sony and Nintendo (BusinessWeek, 2008b; The Washington Post, 2006). At the moment (September 2009) these three companies are the main market players in the hardware market. Microsoft does not sell any PS while the three companies sell competing video game consoles. Sega stopped producing game consoles in 2001 (San Francisco Chronicle, 2001) and Apple and Microsoft are seen as potential new competitors of Sony and Nintendo in the PS market. (BusinessWeek, 2008a; Wall Street Journal, 2006).

2.4.1 Some Basic Figures

In Table 2.1 we report the release dates of the main PS hardware since 1998 for three main markets: North America, Japan and Europe. The release dates for PS seem almost arbitrary and they occur in months that range from February to December for all three regions. However, when we look at the time between releases within companies we discover a different pattern. Table 2.2 shows an average of two-year intervals between releases.

In Table 2.3 we report the release dates on all major VGC since 1987. Clearly, the VGC market is quite different from the PS market. The release dates in North America are mainly chosen to be close to November while in Japan and Europe most releases occur also in other months of the last quarter of the year. If we look at Table 2.4 we can see that there is an additional regularity around the VGC releases. They occur approximately every five years. Only the Sony PS3 took more than 6 years to be released and this was due to a delay in the development of the blu-ray technology added to the PS3. See The New York Times (2006) for more details on this story.

In Table 2.5 and Table 2.6 we report the estimates of single-generation Bass models for PS and VGC. Portable systems have very similar innovation parameters (p) but quite different imitation parameters (q). We computed simple statistics on the Bass models and in most cases they fit the data quite well. We discuss more details on our data next.

2.4.2 Data and Data Cleaning

Our data for the duopoly and triopoly NBM models consists of weekly time series of sales at the USA for the last two PS of Nintendo and Sony and the last two generations of consoles released by Microsoft, Sony and Nintendo. The portable systems are the Nintendo DS, the Nintendo DS Lite, the Sony PlayStation Portable (PSP) and the Sony PSP Slim. The video game consoles are the Microsoft Xbox, Microsoft Xbox 360, Sony PS2, Sony PS3, Nintendo GameCube and Nintendo Wii. In addition, we obtained the corresponding release dates for all products from different news sources and for all cases the release dates matched the date of the first week that we observed in our data. We used a script to download our data from www.vgchartz.com and the site admins authorized us to use their data. Our data for all systems cover the period since their release week up to January 2009. That is, our data covers a period of almost 9 years and 10 systems.

Before we plug our data into the estimation routines we control for indirect network effects, seasonality and price. It has been documented that indirect network effects might play a role in the video game market (see for example, Chintagunta et al. (2009), Clements and Ohashi (2005) or Shankar and Bayus (2003)). Furthermore, Binken and Stremersch (2009) show that it is mainly *super star software* what drives indirect network effects in the video game systems market. Therefore, in this chapter we use a simplified version of the model proposed by Binken and Stremersch (2009) to clean our data from indirect network effects and price. We use the following equation

$$Y_t = \alpha Y_{t-1} + \sum_{j=1...52} \beta_j WD_j + \sum_{l=t...t-L} \lambda_l PCD_l + \sum_{k=t...t-K} \delta_k SSI_k + \epsilon_t \tag{2.22}$$

where Y_t are the system sales at week t; WD_j refers to the week j dummies; PCD_l is the price cut dummies with L total lags and it indicates the week when prices were cut; SSI_k is the total number of *super star software* introduced at week k. To create the independent variables in equation (2.22) we collected release dates and quality ratings on the most popular video games for the systems in our sample. For each system we found approximately 120 video games to construct the SSI variable. In total we collected data for 1200 video games. These data come from many different online sources. Furthermore, we use many different news services to find the price cut timing for all consoles in our sample. We estimated equation (2.22) for each console in our sample and then we subtracted the terms $\sum_{k=t...t-K} \delta_k SSI_k$ and $\sum_{l=t...t-L} \lambda_l PCD_l$ from the consoles sales Y_t only if they are significant. We report in Table 2.7 the sales percentage that indirect network effects represent for each console and the number of lags for the SSI_k variable that we used. We chose the number of lags in the same way as Binken and Stremersch (2009).

Interestingly, despite our model is a much simpler version of that of Binken and Stremersch (2009) we find that indirect network effects represent on average a 13% of the consoles sales while Binken and Stremersch (2009) found that percentage to be 14%. That is, our results confirm their findings. In contrast, we use weekly data, they use monthly, and we find that on average the number of lags correspond to approximately 7 weeks (that is less than 2 months) while they report significant lags up to 5 months. In terms of weeks 5 months represent 20 weeks. We tested lag numbers up to 20 weeks but we did not find significant effects further than 14 weeks (see Table 2.7). Note that the number of lags in the Table should be read with caution because not all lags were found significant and as Binken and Stremersch (2009) we include the last non significant lag to avoid bias. An additional difference is that we estimate the equation

(2.22) separately for each system while they use a panel approach and that their SSI variable is monthly while we trace software introduction per week. Our guess is that they use a panel approach because they consider much shorter time series and the panel approach helped them to identify their model parameters. However, they warn about considerable heterogeneity of the network effects and their result of 14% is therefore close to an average of network effects across systems. Our long time series of weekly data allows us estimate the model for each system and the fit we achieve is very good for all systems (R^2 close to 0.80). A final difference in our approach is that we use the 120 most popular video games per system while they use on average the 10 superstar software video games per system. We estimated a second version of the system models by including only the highly rated video games (the superstars), as do Binken and Stremersch (2009), in the SSI variable. Binken and Stremersch (2009) do not report the percentile they use as a selection heuristic and we selected the video games with a quality rating in the top 25 percentile. In this case, the average network effects jumps up to 15%, while it is also close to their reported number. That is, higher quality video games might have higher network effects although the difference between 13% and 15% can hardly be considered as significant.

The resulting adjusted series without network and price effects still needs to be cleaned from seasonality and for this latter purpose we use the $TRAMO/SEATS$ methodology (Gomez and Maravall, 2001, chap. 8). We further control for all major holidays in the USA and for Easter.

In sum, the series we plug in our estimation routine are the seasonally adjusted series without indirect network and price effects. We use this series because the competitive parameters on our model could pick up the correlation caused by indirect network effects, price and seasonality if we do not control for them.

Our data covers 10 gaming systems and therefore we estimated 20 models (10 for the network effects and 10 for the seasonal adjustment). We do not report these results but they are available from the authors upon request. In addition, we estimate both the duopoly and triopoly models with the original data and the parameter estimates remain very similar. However, the fit is better when we use the clean data.

2.5 Estimation and Parameter Assumptions

We use the systems NLS estimator described in Cameron and Trivedi (2005, Chap. 6, page 217) to estimate the parameters.

The duopoly multi-generation model consists of 26 parameters and in our estimation routine we use 16 free parameters. This number reflects the assumptions that the innovation and imitation coefficient, p and q, vary across firms and products and that the loyalty effects are symmetric. That is, we assume that ϕ_{gk}^{ij} is equal to $-\phi_{kg}^{ji}$. The τ_g^i parameters are the introduction date of each product and we keep the real launch dates in our estimation routine.

The triopoly model consists of 45 parameters and in the estimation routine we have 21 free parameters. This number reflects the assumptions that the p and q parameters vary across firms, that the loyalty parameters are symmetric, and that α_i for $i = x, s, n$ are fixed at some value. The main reduction comes from the assumption that $\phi_{gk}^{ij} = -\phi_{kg}^{ji}$ as it reduces the number of free parameters by 12. Note this is the symmetry assumption we described earlier when we discussed Figure 2.1. Finally, we use the real introduction dates as values for the τ_g^i ($g = 1, 2$ and $i = x, s, n$) parameters.

An important assumption in the estimation routine is the value of the α parameters and we need an assumption on them. As we mentioned earlier, the α parameter is simply the share of the sales that the first generation transfers to the second generation. The reason why we need to make an assumption regarding α is that there is a direct relationship between the α and the m parameters with the realized cumulative sales. We know that the realized cumulative market sales are fixed at some value, call it M, and it depends on both α and m. Of course, the realized M depends on all other parameters but specially the α and the m are very closely related to it. If we increase α then we need a lower m to keep the realized sales at M or if we lower α we need a higher m. This means that we can not simultaneously identify both parameters. This is a limitation and at the same time an advantage of our model because we can obtain the α parameter easily from experts opinions, managers, store sales data, or surveys. All we need to know is what percentage of the first generation sales (of an specific firm) is transferred to its second generation and that is α. However, in case the α is not available then we could make assumptions on the market potentials and estimate the α together with all other free parameters in the model. We know that market potential assumptions are quite common in the new products diffusion literature and they are straightforward to construct.

In the estimation routine first we assume the $\alpha = 1$ for all firms in both the duopoly and triopoly model. Then, as an illustration, we ask an expert opinion on the size of α for each firm in our triopoly model. We contacted a local store manager and asked him about the α parameter of Microsoft, Sony and Nintendo according to his experience. His information is that the α of Microsoft is 0.3, the α of Sony is 0.1 and the α of Nintendo is 1.1. These numbers imply that Nintendo is able to get 1.1 sold unit of Wii for each sold unit of the GameCube, Sony achieves the lowest with a 0.1 of PS2 unit sales going into the PS3, while Microsoft is in between with an α of 0.3.

To estimate both models we use the systems NLS estimator but due to the large number of parameters we split estimation in three steps. First we estimate the six innovation and imitation coefficients p and q given all other parameters fixed. Next we estimate the loyalty coefficients ϕ given all other parameters are fixed at their most recent estimated values. We iterate these two steps until convergence and at the end of the routine we estimate the six market potentials given all other parameters. Chintagunta et al. (2009) apply a similar estimation approach. In the estimation routine we constrained the ϕ coefficients setting their lower and upper limits at -4 and $+4$, respectively. However, all parameter estimates are within these limits as we report in Section 3.5. All our routines are programmed in R (R Development Core Team, 2005).

2.6 Estimation Results

We report the parameter estimates for the duopoly model in Table 2.8. In this model we consider two companies, Nintendo and Sony, and their portable gaming systems. The systems are the Nintendo DS and DS Lite and the Sony PlayStation Portable (PSP) and PSP Slim. We notice that the parameter estimates for the innovation and imitation parameters, p and q, are lower in the multi-generation model than in the independent Bass model reported in Table 2.5. In addition, the market potentials are remarkably lower in the multi-generation model. Two factors explain the lower estimates. First, the multi-generation model allows the first generation to transfer a percentage α of its sales to the second generation. Hence, the second generation market potential has a lower m estimate but note that the realized market potential in the multi-generation model may be higher than the m estimate after adding the competition and cannibalization effects. These results are in line with the findings of Norton and Bass (1987) regarding the size of the market potentials of the second generation products; see (Norton and Bass, 1987, footnote 2, page 1074). Finally, we find significant ϕ parameters and this is evidence supporting the idea that the portable systems compete against each other. For example, we see that the Nintendo DS is losing share to the Sony PSP (see the -0.57 estimate) and it is losing more to the second generation of Sony, the PSP Slim (see the -2.39 estimate). On the other hand, the Nintendo DS Lite is receiving a share from the PSP Slim (see the 0.66 estimate). We report the model fit in Figure 2.3 and we can see the fit is reasonably good.

In Table 2.9 we report the triopoly model parameter estimates with the assumption that all firm's $\alpha = 1$. In Table 2.10 we present the parameter estimates when we use $0.3, 0.1$ and 1.1 as the α parameters for Microsoft, Sony and Nintendo, respectively. Finally, in Table 2.11 we present the ϕ and α parameters reported in Table 2.9 in a easy to read format.

For the triopoly case it is the q parameter estimates that are much lower than in the Bass model reported in Table 2.6 while the p parameters remain very similar. An interesting result is that the Microsoft Xbox market potential is around 19 million units while the Xbox 360 market potential is a much lower value of 813 thousand units. A similar drop in market potential occurs from the Sony PS2 to the Sony PS3. The exception is Nintendo. The market potentials for both the Nintendo GameCube and the Nintendo Wii stay around the same level (17 million units). This finding is in line with the results of Shankar and Bayus (2003). Shankar and Bayus (2003) analyze the video game market between 1993 and 1995 and the two main players at that time where Nintendo and Sega. Note that in Table 2.3 we report the history of console releases since 1985 and that they analyzed the last three years of the 4th generation systems. They argue that Nintendo had a higher network strength than Sega and consequently Nintendo sales overtook those of Sega. Recently, the Nintendo Wii is overtaking the sales of the largest player, Sony, and our parameter estimates seem to capture this overtake.

In Table 2.10 we report the model with our expert's values on the α parameters. As we anticipated, the parameter estimates of the market potential m are higher for the second generation of Microsoft and Sony because we assumed a much lower α for them (0.1 and 0.3, respectively). The market potential for the Xbox360 goes from 813 thousand units in the first model up to $2,685$ thousand units in the second, that is 3.3 times higher. The PS3 m in the second model is 2.14 times higher than in the first. Finally, the market potential of Nintendo's second generation, the Wii, is 1.161 million units lower in the second model relative to the first because of the higher α. Surprisingly, the market potential of both generations of Nintendo are still high relative to each other despite the fact that Nintendo can transfer more consumers from the GameCube to the Wii (it has the highest α among the three companies). The rest of the parameters in Table 2.10, with very few exceptions, remain very close to the model parameters of Table 2.9. We are certain that there are other ways to retrieve the α parameters from experts, surveys or data we stress that this estimation exercise is just an illustration.

In Table 2.11 we arrange the ϕ and α parameters in two six by six tables. We numbered the estimated ϕ parameters in the top table and in the bottom we report their estimates using bold face for parameters with t-values higher than 1. We can see that the Wii is getting some share from the Xbox console (see the 2.39 parameter of the phi[4]) and that is is not competing against the PS3 (see the -0.02 of the phi[12]). This confirms what has been argued in the press that these two consoles are not substitutes for each other. A surprising result is that the Wii has a positive influence on PS2 (see the -0.60 estimate of the phi[8]). The PS3 is losing some share to the Xbox 360 and the GameCube (see the phi[6] and phi[11] estimates) according to the sign of the parameters but they are not significant. At the same time, the PS2 received share from the Xbox 360 and the GameCube. Most parameter estimates are in line with our anecdotal evidence and what we read in the press.

Finally, we plot the observed and fitted values of the triopoly model in Figure 2.4 and again the model fits the data reasonably well. Note that the real cumulative sales of the first generation products, the graphs in the left of Figure 2.4, stabilize after they reach their maximum. However, our model forecasts a decline in their number of cumulative units after reaching the maximum and this is a consequence of the substitution that takes place after new generations are introduced. Hence, the fit after the maximum is not really the same as the fit before the maximum of the cumulative sales given that we do not have data on substitution or *un-adoption* of these products. An interesting feature of the left-hand graphs is that the foreseen decline is faster for the Xbox and the GameCube while it is very slow for the PS2.

2.7 Duopoly Case Study: The Portable System Race

In this section we use our model to analyze the portable system market. We take the duopoly model and its parameter estimates and with them we simulate four different strategies for both Nintendo and Sony. We use a planning horizon $T_p = 90$ months and this number is long enough relative to the average pace of two years we report in Table 2.2. Next we describe the strategies we simulate and afterwards we present the insights gained by our numerical exercises. At the end of the section we present the sensitivity analysis to different parameter estimates.

2.7.1 Simulating Plausible Strategies

A *strategy* is a complete contingent plan for all market players. (Watson, 2002, pg. 26). That is, we define the actions of Nintendo as a response to any of Sony's actions and viceversa. In all of the strategies, except the first, we let Nintendo be the *leader* and Sony the *follower*. We reversed their roles in our numerical exercises and our insights remain without significant changes. Furthermore, the *leader-follower* assumption is common in the literature, see for example Bayus et al. (1997, p. 56). Finally, we assume that the order of entry does not modify the competitive relationship between products, just as in Kamien and Schwartz (1972), but note that we will provide sensitivity analysis to different parameter values in the next section.

The four strategies we consider are:

1. **Random Date Selection**: In this strategy both Nintendo and Sony randomly select a launch date for their two product generations at the beginning of the planning horizon. That is, both firms ignore each other's actions and the interaction among their competing products.

2. **Imitation**: In this strategy Nintendo selects the launch-timing for its two generation products and Sony imitates Nintendo. That is, Sony launches its PS2 console at the same time as the GameCube and it launches the PS3 at the same time as the Nintendo Wii.

3. **Pre-commitment and Optimization**: In this strategy Nintendo pre-commits to the launch date of their two generation products while Sony, with perfect foresight, optimizes the launch dates of its two generation products based on Nintendo pre-commitment dates.

4. **Uncertain Dates and Stochastic Optimization**: In this strategy Nintendo does not pre-commit to a launch date for its two generation products. However, Sony assigns a probability to each of the possible launch-timings of the GameCube and the Wii and based on this information it optimizes the launch-timing of the PS2 and the PS3.

We give the details of each strategy in the Appendix 2.A. We simulate these four strategies and we compute the outcome in terms of the maximum cumulative sales of Sony, Nintendo and the sum of both firms' maximum cumulative sales. We repeat the simulation of each strategy until we cover all the combinations possible of the launch-timing selected by Sony and Nintendo that each strategy implies. In this way we recover the distribution of the sales that both players may achieve by following each of the four strategies. We summarize these distributions in Table 2.12 and Figure 2.5.

In Table 2.12 we report six quantiles of the distribution of the sales for Sony, Nintendo and their sum and for each of the four strategies while in Figure 2.5 we plot their percentiles. The purpose of Table 2.12 and Figure 2.5 is to help us rank the strategies in terms of the likelihood of their sales outcomes. For example, in Table 2.12 we see that for Sony the sales achieved by imitating are lower than the sales achieved by randomly selecting its dates, see the second and fourth lines in the table.

In the right-hand side of Figure 2.5 we see that the strategy that results in higher sales for Sony is the third and that is the strategy in which Sony knows the exact launch dates of Nintendo's products. Only at the very first percentiles (from 0 to around 20%) the stochastic optimization strategy is better. In the graph it is clear that the second best strategy results when Sony applies stochastic optimization. As we can notice, this strategy puts a lower and upper limit to the sales of Sony, see the flat areas of the *uncertain dates* line at the first and last percentiles. Surprisingly, imitation is the worst strategy Sony could follow and it performs slightly worse than when Sony randomly selects its dates.

In the left-hand side of Figure 2.5 we see the quantiles of the distribution of sales achieved by Nintendo. Note that Nintendo is the leader and the outcomes are therefore not a mirror of the results obtained by Sony. For Nintendo the results are mixed. We see that before the percentile 50 the best outcome is achieved when Sony is imitating (interestingly this is not a good option for Sony) and that after the 50 percentile the best outcome is achieved by not announcing its launch dates and by not precommiting to them (see the *uncertain dates* line). On the other hand, before the 50 percentile the lowest sales are achieved when Sony uses stochastic optimization and above the 50% the lowest sales are either random selection of dates or pre-commitment. Note that Nintendo does not behave strategically in our simulations. That is, Nintendo does not know that Sony is following one of the four strategies. Given that Nintendo knows which strategy Sony is playing then it is straightforward for Nintendo to strategically select its launch dates and achieve high sales. This implies that if Nintendo strategically chooses its launch dates then playing the *uncertain dates* strategy can result in high sales while if Nintendo acts not strategically then pre-commitment is a reasonable strategy. Of course, we are not using very strict criteria to rank Nintendo's strategies but it is straightforward to rank the strategies using different criteria given we know their corresponding outcomes in terms of sales distributions.

2.7.2 Sensitivity Analysis of the Launch Strategies

The above results are sensitive to the parameter values we plug in the duopoly model. In all previous exercises we used the values we obtained from our estimation routine. To know how the sales outcome may change we compute the expected value of the sales achieved by playing the second strategy (imitation) and the third strategy (optimization) when we plug in a different set of parameter values in the model. First we evaluate the strategy by simulating different combinations for the phi[1] and phi[2] parameters, the phi[3] and phi[4] parameters and finally for the phi[1] and the alpha parameter of Nintendo. The phi[1] and phi[2] are the ϕ parameters between the Nintendo DS and the PSP and the PSP Slim, respectively. The phi[3] and phi[4] are the ϕ parameters between the Nintendo DS Lite and the PSP and the PSP Slim, respectively.

In Figure 2.6 we report the log of the ratio of the expected sales of Nintendo and Sony given all possible combinations of these parameters, take two at a time, for the imitation and pre-commitment strategies. In the ratio Nintendo's expected sales is the numerator. This is a numerical intensive exercise in the sense that for each parameter combination we compute all possible combinations of launch-timings implied by each strategy and based on the outcome (in terms of their maximum cumulative sales) we compute the expected value for the sales of both players. In the graphs we report the log of the ratio of the expected maximum cumulative sales between the two firms. Note that we apply the log transformation to the final values because the log of the expected value is not the same as the expected value of the logs.

The graphs in Figure 2.6 provide a unifying message. Both strategies might yield high sales if a firm's products are superior (in terms of the ϕ) parameters or if a firm's ability to transfer users of old technologies to new ones is high (that is equivalent to a high alpha). If both ϕ parameters tend to be positive the ratio goes up and therefore Nintendo sells more relative to Sony. The ratio increases in a similar way when the alpha of Nintendo is higher. Earlier we concluded that the imitation strategy is the worst among the four strategies we evaluated for Sony. However, if Sony had superior products the imitation strategy may yield high sales, see how the log ratio goes up to -3 and -2 in the left-most and center upper panel graphs. This is evidence supporting Lemma 2. However, we can easily notice that despite the unifying message the surfaces have different slopes. That is, achieving higher sales by raising or decreasing each of the ϕ parameters does not yield the same increase/decrease in expected sales. We conducted the same sensitivity analysis for the random dates and the stochastic optimization strategies and the results are very similar.

The main lesson of this sensitivity analysis is that the outcome of any launch-timing strategy varies radically and it depends heavily on the competitive positioning of the firms' products and on the firms' ability to transfer users of their old technologies to the new ones.

2.8 Triopoly Case Study: The Video Game Console Race

In this section we present a different set of numerical sensitivity analyses and we will focus on the launch-timing of the Sony PlayStation 3 and the Nintendo Wii relative to their previous generations and relative to their competitors. In this section we focus on answering *what if* questions rather than studying the strategic interaction of firms, like in the previous subsection. We use the parameters estimates we obtained from our estimation routine to answer the *what if* questions and we assume a planning horizon $T_p = 150$ months. That is we assume 12.5 years as planning horizon and this is in line with a recent interview statement of the President of Sony Computer Entertainment in America, see Fast Company Blog (2009). In addition, we illustrate the sensitivity of the optimal launch-timing to different competitive and cannibalization parameters.

In Table 2.3 we reported the release dates of all major video game systems. It easy to notice that historically the phenomena of *a launch race* in a single year is relatively a recent experience for system manufacturers. This is interesting given that the number of systems manufacturers has stayed relatively constant since the early nineties. We observed for example that the Nintendo Wii was launched at the same time as the PlayStation 3 in North America three years ago. The GameCube and the Xbox were launched simultaneously in 2001. The other close to simultaneous launch cases occurred between the Wii and PS3 in Japan and between the Xbox and GameCube in Europe in 2006 and 2002, respectively. The average timing between releases is approximately five years (5.09 years), and the standard deviation of this average is almost one year (0.90 years), see Table 2.4. Hence, we believe that there is a need for insights about whether these launch-timing were chosen optimally or what could make them optimal.

The optimization situations that we consider next are much simpler than the optimization situations that we encounter in practice. They are simpler because of mainly two reasons. First, we do not consider the strategic interaction between firms as in previous section. Second, we do not consider price as part of the optimization problem because we focus on analyzing the launch-timing decision relative to different cannibalization and competitive settings. However, the timing decision can be considered as a sub-game of the price and timing game. That is, our analysis has no assumption regarding the price of the consoles and we focus on the effects of timing dates on the unit sales of the systems. This assumption is in line with similar studies to ours, see for example Joshi et al. (2009) and the work cited by (Souza et al., 2004, p. 538) regarding pricing assumptions. However, we do not consider this a very strong assumption in terms of our model estimation because of the cleaning procedure of our data. Nonetheless, if we had a reasonable assumption about the price for all six systems in our triopoly or duopoly model, and how the prices of all systems are strategically related to each other, then it is straightforward to introduce it in the optimization problem. Still, our results will be valid as price would possibly work as a discounting factor in the optimization problem. Of course, the effect of price on demand is not a straightforward introduction into our diffusion model and we consider this an area of further research.

2.8.1 Simulating *What If* Questions

The first *what if* question we answer is: What would be the maximum cumulative sales of the Nintendo and Sony if they would have launched their consoles at different dates and leaving everything else constant? That is, we answer how either the sum of the maximum of equation (2.12) and (2.13) for Sony and the sum of the maximum of equation (2.14) and (2.15) for Nintendo are maximized. In Figure 2.7 we plot the total sales of Nintendo (summing up the maximum cumulative sales of the Wii and the GameCube) achieved by launching at different dates. The maximum cumulative sales are reached when the Wii is launched at the month 64 (that is April 2005) and the GameCube at month 1 (January 2000). That is 5.33 years between their releases. The real release time between these two consoles was 5.01 years in North America, 5.22 years in Japan and 4.60 in Europe. The real launch dates happened at November 2006 (month 83 in the graph) and November 2001 (month 23 in the graph). Surprisingly, Nintendo is not launching that far from the optimal dates and according to this surface the difference of sales between real and optimal dates is $3,858.62$ thousand units ($66,431.66$ thousand units at the optimal and $62,573.04$ at their real launch dates). The story is different for Sony. In Figure 2.8 the maximum is reached when the PS2 is launched at month 1 (January 2000) and with the PS3 not launched. Note that setting the month of launch equal to the end of the planning horizon is equivalent to not launching. This is a radical scenario but it is explained by the fact that the PS2 is receiving sales from both the Xbox 360 and the Nintendo Wii according to our model estimates while the PS3 competitive parameters are not very favorable, see Table 2.11. The real launch dates of the PS2 and PS3 are the months 10 (October, 2000) and 84 (December, 2006), respectively. The total sales of Sony at these last pair of dates is 59.988 million units, in Figure 2.8 all the sales surface is graphed for all possible launch dates. We know that up to the first week of August 2009 the PS2 has sold 50.767 million units (source vgchartz.com). Hence according to our model the realized sales of PS3 will be around 9.22 ($\pm$2.14) million units while up to date the Sony PS3 has sold 9.018 million units. The 2.14 million units is the average derivative of the surface at the real launch dates, the point $(10, 84)$ in Figure 2.8. Therefore, our model is not very optimistic about the PS3.

The next questions we answer are: what is the optimal launch time of the Nintendo Wii given the launch times of the Sony PS3? and what is the optimal time of the Sony PS3 given the launch times of the Wii? We can answer these questions by looking at Figure 2.9. In this figure we present two contour graphs (or heat maps). The lighter (yellow) areas represent higher total sales and the darker (red) areas represent lower sales. We call these graphs *sales reaction surfaces* because we can derive the best reaction function of either Nintendo or Sony given each other introduction timings. A *reaction function* maps any launch-timing of a firm to the best launch-timing of a second firm. We use the same definition of *reaction functions* as in Section 2.7. For example, in the left-hand graph we see that the maximum of Nintendo's sales is on month 73 given Sony launched its PS3 in month 1. From Table 2.11 we know that the PS3 and the Wii are not close competitors and not surprisingly the optimal launch date of the Wii given any introduction date of the PS3 remains close to the month 73 (January 2006) for any introduction timing of the PS3. What is surprising is that Nintendo launched 11 months later than its optimal timing. In the right-hand graph we see that the optimal launch dates of Sony are not very sensitive to those of the Nintendo Wii. For example, if the Wii were launched from month 1 up to the month 60 (that is from January 2000 up to December 2004) then the optimal month for the PS3 remains very close to the month 126 (June 2010). However, if the Wii is launched after the month 80 then the optimal action for Sony is to set the introduction date of the PS3 at month 150, the end of the planning horizon. Hence, the best strategy for Sony if the Wii is launched after month 80, is not to launch the PS3.

2.8.2 Sensitivity Analysis of the Optimal Launch-Timing

In the previous subsection we answered *what if* questions assuming our model parameter values are the ones resulting from the estimation routine. However, the optimal timing is sensitive to the parameter values and in this subsection we present how sensitive it is to different competitive and cannibalization settings.

First we present the sensitivity of the optimal launch date of the Sony PS3 to the competitive parameters that relate this console to the Xbox 360 and the Wii, the phi[6] and phi[12] respectively, for six different scenarios. In each of these scenarios we assume an early, a late, and an intermediate introduction timing of the Xbox 360 and the Wii. That is, we present three scenarios for each last generation console that competes against the PS3. Second, we present the sensitivity of the optimal launch date of the Sony PS3 given different cannibalization and competitive parameters using these same six possible scenarios. We present these results in Figure 2.10 and Figure 2.11 respectively.

In Figure 2.10 we present the scenarios for early (month 40), intermediate (month 84) and late (month 120) introduction timings of the Microsoft Xbox 360 at the upper graphs. In the graphs at the bottom we present the scenarios with the Nintendo Wii launched at the same set of introduction timings. For all six scenarios we leave all other introduction timings and parameters at their real or estimated values, respectively. Note that we only use the ϕ parameters that relate the three systems in our scenarios and set the others at their estimated values.

The first lesson we derive from Figure 2.10 is that the optimal timing of the PS3 depends on how it is competitively related to its two main competitors and not to only one of them. The second insight is that there is a parameter space for which it is better not to launch the PS3 (that is the flat top area in all graphs). Therefore, we can visualize the parameter space where Lemma 3 holds, these are the flat top *don't launch areas* in Figure 2.10. Hence, the *launch never* might be optimal depending on the competitive positioning of the PS3. Similarly, there is a parameter space for which there are earlier optimal introduction timings for the PS3. The third insight is that, the parameter space that is suitable for an earlier introduction gets reduced when the competing consoles are launched at later stages. See how the flat surface (the *don't launch area*) is larger for the center and right graphs relative to the left most graph. The fourth insight is that even when the competitive parameters are very favorable for the Sony PS3, its earliest optimal introduction timing happens at the month 60 (December 2004) and that would imply a 4.16 years difference between the PS2 and the PS3. That is, the *launch now* solution is not part of a very favorable set of parameter values. Note that this time difference between consoles is on the low side of the time between actual releases for all the major video game systems reported in Table 2.4.

This last result may point that the 4 year time between releases could be a good introduction pacing strategy when the product is superior relative to its competitors. Interestingly, the time between releases are in the low side for third and fourth generation consoles and they are in the high side for the the six and seventh generation systems. We do not have data on the earlier systems but our intuition is that the fourth generation consoles were superior to the third generation consoles and they were better positioned relative to its competitors. This may be the case, for example, of the Sega Genesis and the Sega Dreamcast launched 4.33 and 3.25 years after their previous generation, respectively. According to our discussions with some hard-core gamers that seems to have been the case indeed. In contrast, we have read in the press that the relative positioning of the Sony PS3 and the Xbox 360, for example, is not very strong relative to each other and this coincides both with longer time between releases diagnosed by our model and with the longer time between releases we document in Table 2.4 for the latest product generations.

In Figure 2.11 we present the sensitivity analysis of the optimal launch-timing of the PS3 to different cannibalization and competitive parameters, that is concerning the α and ϕ parameters. The upper graphs show the optimal timing of the PS3 for three scenarios of the launch-timing of the Xbox 360, similar as previous graphs. In the bottom graphs we present the scenarios with different introduction timings of the Nintendo Wii. The main difference between this and the previous figure is that one of the axis is now replaced by Sony's *alpha*. In the upper graphs we consider the cannibalization parameter of Sony and the ϕ parameter (phi[6]) that relates the Xbox 360 and the PS3. In the bottom graphs we use the same cannibalization parameter of Sony and the ϕ parameter that relates the Wii and the PS3, the phi[12]. The range we use for the α cannibalization goes from 0 up to 3. A higher number than 1 would imply that Sony is able to get more than one unit sale of the PS3 for each PS2 sold.

The first insight we derive from Figure 2.11 is that the optimal introduction timing of the PS3 depends on both the relative positioning to its competitors and to the cannibalization between Sony's generations. The second insight is that, as before, there is a parameter space for which it is optimal not to launch the PS3 (the top flat *don't launch areas*) and this space seems larger when competitors launch their consoles at late introduction dates. The third new insight is that the larger Sony's α is, the sooner it is optimal to introduce the PS3. If there is little cannibalization, for example for α values between 0 and 0.5, then it is optimal for Sony to set the launch-timing of the PS3 closer to the end of the planning horizon. For example, in the leftmost bottom graph the optimal timing for a low α values ranges between the month 100 (April 2008) and 129 (January 2010), when the phi[12] value is equal to 2. However, if the α value is larger (near 3 in the same graph) the optimal timing stabilizes at 81 (September 2006). The middle bottom graph corresponds to the scenario that considers the real introduction date for the Wii and in this graph the optimal timing stabilizes at month 65 (May 2005) when both the phi[12] and the α parameter are very favorable to Sony. The optimal timing stabilizes in all graphs around the month 64 (April 2005) and this month implies 4.5 years between releases. Therefore, the *launch now* strategy is not a result of very favorable competitive and *alpha* parameters. The real launch of the PS3 occurred in month 84 and this month is optimal only when the α is much larger than 1 and with a phi[12] approximately near 1. This may indicate that Sony's management might have been very optimistic about the PS3 when they chose that month, at least according to our model.

Finally, the last insight is that when there is no cannibalization the optimal timing of the PS3 is at time 0, that is the *launch now* strategy is covered only as a special case when there is no cannibalization between generations, (Lemma 1). In all the graphs of Figure 2.11 we can see that the *don't launch* area does not reach the $\alpha = 0$ and at this parameter value the optimal timing drops rapidly to the very start of the planning horizon. See the little empty space between the *don't launch area* and the back wall in all graphs. Visually, it is easier to detect how the surface drops to zero in the upper graphs.

To summarize, the *launch now* strategy results only when there is no cannibalization between a firm's product while the *launch never* strategy results when there are late product introductions by competitors, when a *firm's alpha* is very low, or when the competition is intense in terms of the ϕ parameters. In addition, we find that very favorable competitive and alpha parameters do not imply the *launch now* strategy as we discovered that the optimal launch-timing seems to reach a limit of 4 years between generations. Finally, we find that the higher a firm's ability to transfer its old technologies users to the new ones, the earlier it is optimal for it to introduce new generation products.

2.9 Conclusions and Discussion

In this chapter we presented a new model that is helpful to analyze different launch-timing strategies and optimal introduction timings. It is straightforward to estimate the model parameters and to analyze different interesting competitive and *firms' alpha* scenarios. Our model is suitable to study settings where there are just a few market players or products and when there are some dominant *alpha* technologies in the market.

The insights we gained is that the *launch now or never* strategies may arise depending on the competitive parameters and the relationship between the products in the market. Specifically, the *launch now or never* strategies arise when there are late product introductions by competitors, when a *firm's alpha* is very low, or when competition is intense.

For the first time in the academic literature we provide some insights into the introduction strategy of the main players in the studied industry and we document their introductions since the late eighties. We find that the launch strategy of each 4 years seems appropriate when there is a better product positioning or very high *alphas*. That seemed to be the case at the early stages of the game systems industry while it is not any more so now.

According to our model, Nintendo launched the Wii at an appropriate moment while the Sony PS3 perhaps should have never been launched. Moreover, we find that different strategic interactions between firms lead to different sales levels and we argue that the strategy should be chosen relative to the firms' *alpha* and relative to the competitive setting that its products face. For example, the imitation strategy returns are higher for certain competitive parameters, specifically when the product is superior.

The managerial implications are clear. According to our insights the managers in industries with *alpha* technologies should pay not only attention to the competition but also to the ability of their firms to transfer users of old technologies to new ones. In our case study we pointed out that the outlook for PS3 is not very promising, it may reach maximum a 12 million unit sales according to our estimates. However, if Sony's managers work in new ways to increase Sony's *alpha* or its competitive positioning the outlook for the PS3 could improve.

The higher a firm's ability to transfer its old technologies users to the new ones, the earlier it is optimal for it to introduce new generation products. Think of the situation where the first generation product of a firm may face stiff competition after a point in time while its second generation is better equipped to fight against the new entrant. In this scenario, the best and perhaps the only surviving strategy would be to transfer its users of old technologies to the new ones as soon as possible and before competitive entry. We speculate then that the ability to survive in such market depends partially but heavily on the *firm's alphas*.

In our view, the technology markets mimic some of the competitive behavior of the *alpha* chimpanzees. The *alpha* rank for a chimpanzee means access to desirable foods, females or resting places while for companies the *alpha* rank means access to the users of their own old technologies. However, note that in the chapter we assumed non-cooperative behavior between firms while it has been documented that *alpha males* in the chimpanzee society may form temporal alliances to overcome the current dominant *alpha male* (Nishida, 1983). This is a situation we do not study and that we may encounter in the future of the game systems markets. For example, the recent search alliance between Yahoo and Microsoft and the alliance between Toshiba and Sony regarding the blu-ray standard seem to be in line with the cooperative behavior of chimpanzees reported by Nishida (1983). On the other hand, the potential entrance of Apple and Microsoft in the portable gaming systems market points towards the arrival of more *alpha technologies* and hence perhaps more competition. Finally, we left out other aspects of the marketing mix that may prove important in the timing of new dominant technologies. We consider all these extensions interesting avenues for further research.

2.10 Tables and Figures

Firm	Portable System	North America	Japan	Europe
Nintendo	**DS Lite**	June 11, 2006	March 2, 2006	June 23, 2006
	DS	November 21, 2004	December 2, 2004	March 11, 2005
	GameBoy Advance SP	February 15, 2003	February 14, 2003	March 28, 2003
	GameBoy Advance	June 11, 2001	March 21, 2001	June 22, 2001
	GameBoy Color	November 19, 1998	October 21, 1998	November 23, 1998
	GameBoy	August 15, 1989	April 21, 1989	1990
Sony	**PSP Slim Lite**	September 5, 2007	September 13, 2007	September 5, 2007
	PSP	March 24, 2005	December 12, 2004	September 1, 2005

Source: VGchartz, Wikipedia & online press articles. Notes: We report the year of introduction when the exact date is not available.

Table 2.1: Release Dates of Portable Systems

Firm	Transition to/from	North America	Japan	Europe
Nintendo	**DS - DSLite**	1.55	1.25	1.28
	GBA SP - DS	1.77	1.80	1.96
	GBA - GBA SP	1.68	1.90	1.76
	GBC - GBA	2.56	2.42	2.58
	GB - GBC	9.27	9.51	–
Sony	**PSP Slim - PSP**	2.45	2.75	2.01

Table 2.2: Release Time Between Portable Systems (in Years)

Generation	Firm	Console	North America	Japan	Europe
7th generation	**Nintendo**	**Wii**	November 19, 2006	December 2, 2006	December 8, 2006
	Sony	**PlayStation 3**	November 17, 2006	November 11, 2006	March 23, 2007
	Microsoft	**Xbox 360**	November 22, 2005	December 10, 2005	December 2, 2005
6th generation	**Nintendo**	**GameCube**	November 18, 2001	September 14, 2001	May 3, 2002
	Sony	**PlayStation 2**	October 26, 2000	March 4, 2000	November 24, 2000
	Microsoft	**Xbox**	November 15, 2001	February 22, 2002	March 14, 2002
	Sega	**Dreamcast**	September 9, 1999	November 27, 1998	October 14, 1999
5th generation	**Nintendo**	**N64**	September 29, 1996	June 29, 1996	March 1, 1997
	Sony	**PlayStation**	September 9, 1995	December 3, 1994	September 29, 1995
	Sega	**Saturn**	May 11, 1995	November 22, 1994	July 8, 1995
	Atari	**Jaguar**	November 18, 1993	–	–
4th generation	**Nintendo**	**Super Nintendo**	August 13, 1991	November 21, 1990	April 11, 1992
	Sega	**Genesis**	September 15, 1989	October 29, 1988	November 30, 1990
3rd generation	**Nintendo**	**Nintendo**	October 18, 1985	July 15, 1983	–
	Sega	**Master System**	June 15, 1986	1985	1987

Source: VGChartz, Wikipedia & online press articles. Notes: We report the year of introduction when the exact date is not available.

Table 2.3: Release Dates of Major Video Game Consoles

Firm	Transition to/from	North America	Japan	Europe
Nintendo	**Wii - GameCube**	5.01	5.22	4.60
	GameCube -N64	5.14	5.21	5.18
	N64 - SNES	5.13	5.61	4.89
	SNES - Nintendo	5.82	7.36	–
Sony	**PS3 - PS2**	6.06	6.69	6.33
	PS2 - PS1	5.13	5.25	5.16
Microsoft	**Xbox 360 - Xbox**	4.02	3.80	3.72
Sega	**Dreamcast - Saturn**	4.33	4.02	4.27
	Saturn - Genesis	5.65	6.07	4.61
	Genesis - Master Sys	3.25	–	–

Table 2.4: Time Between Major VGC Releases (in years).

Video Game Console	m (thousand units)	p	q	Sample
Nintendo DS	**6799.3447****	**0.0140****	**0.1789****	**Nov 2004 - June 08**
	(1975.3660)	(0.0056)	(0.0543)	
Nintendo DS Lite	**27972.9479****	**0.0403****	**1.9922****	**June 2006 - Jan 2009**
	(1130.8544)	(0.0146)	(0.2999)	
PSP	**9717.9772****	**0.0109****	**0.1500****	**Mar 2005 - Sep 2007**
	(1525.7210)	(0.0026)	(0.0389)	
PSP Slim Lite	**7068.5424****	**0.0184***	**0.2449***	**Sep 2007 - Jan 2009**
	(2579.1973)	(0.0091)	(0.1168)	

Note: standard error in parentheses; *,** mean that the coefficient is significant with 95% and 99% confidence respectively

Table 2.5: Bass Model Estimates for Portable Systems

Video Game Console	m	p	q	Sample
Xbox	**16157.4500****	**0.0058****	**0.0993****	**Nov 2001 - Aug 2007**
	(699.5485)	(0.0009)	(0.0132)	
Xbox 360	**16312.2600****	**0.0054****	**0.1272****	**Nov 2005 - Jan 2009**
	(2826.5520)	(0.0012)	(0.0304)	
PlayStation2	**47847.1300****	**0.0037****	**0.0619****	**Oct 2000 - Jan 2009**
	(4520.1510)	(0.0007)	(0.0149)	
PlayStation3	**8190.0120****	**0.0075****	**0.1789****	**Nov 2006 - Jan 2009**
	(1173.5730)	(0.0014)	(0.0333)	
GameCube	**12716.7600****	**0.0058****	**0.0959****	**Nov 2001 - Apr 2008**
	(527.1293)	(0.0009)	(0.0142)	
Wii	**23353.9300****	**0.0063****	**0.1672****	**Nov 2006 - Jan 2009**
	(4673.3370)	(0.0014)	(0.0340)	

Note: standard error in parentheses; ** mean that coefficients are significant with 99% confidence.

Table 2.6: Bass Model Estimates for Video Game Consoles

System	**Model Lags**	**% Network Effects**
Nintendo GameCube	4	15.77%
Nintendo Wii	11	23.33%
Sony PlayStation2	–	–
Sony PlayStation3	5	2.28%
Microsoft Xbox	3	3.60%
Microsoft Xbox 360	7	6.69%
Nintendo DS	14	37.24%
Sony PSP	9	2.28%
All Systems	**7.57**	**13.03%**

Table 2.7: Video Game Effects on Game Systems

Coefficient	System	Estimate	s.e.	t-value
p	**DS**	**0.01189**	(0.0018)	6.56
	DS Lite	**0.03243**	(0.0040)	8.12
	PSP	**0.05556**	(0.0106)	5.25
	PSP Slim	**0.02843**	(0.0076)	3.74
q	**DS**	**0.07897**	(0.0406)	1.94
	DS Lite	**0.08174**	(0.0353)	2.31
	PSP	**-0.08717**	(0.0918)	-0.95
	PSP Slim	**0.12453**	(0.0358)	3.48
phis	**(1)**	**-0.57882**	(0.6478)	-0.89
	(2)	**-2.39516**	(1.9112)	-1.25
	(3)	**-0.41831**	(0.2109)	-1.98
	(4)	**0.66457**	(0.2196)	3.03
m	**DS**	**15991.8**	(530.0378)	30.17
	DS Lite	**10980.7**	(486.4696)	22.57
	PSP	**10498.0**	(169.4519)	61.95
	PSP Slim	**1012.0**	(205.1027)	4.93

Note: phis (1) is the substitution coefficient between DS and PSP, phi(2) between DS and PSP Slim, phi(3) between DS Lite and PSP, and phi(4) between DS Lite and PSP Slim; s.e. stands for standard error.

Table 2.8: Multi-Generation Model for Portable Systems

Coefficient	Console	Estimate	s.e.	t-value
p	**Microsoft**	**0.00943**	(0.0025)	3.79
	Sony	**0.00980**	(0.0011)	8.56
	Nintendo	**0.01156**	(0.0021)	5.63
q	**Microsoft**	**0.06115**	(0.0143)	4.27
	Sony	**0.03881**	(0.0051)	7.55
	Nintendo	**0.05381**	(0.0144)	3.74
phis	[**1**]	**-0.00512**	(0.2087)	-0.02
	[**2**]	**-0.07195**	(0.8990)	-0.08
	[**3**]	**0.02003**	(0.2270)	0.09
	[**4**]	**-2.39045**	(1.0587)	-2.26
	[**5**]	**-0.28843**	(0.1261)	-2.29
	[**6**]	**0.31218**	(0.8135)	0.38
	[**7**]	**0.62241**	(0.2734)	2.28
	[**8**]	**-0.20223**	(0.6419)	-0.32
	[**9**]	**0.11293**	(0.2238)	0.50
	[**10**]	**0.60116**	(0.1576)	3.81
	[**11**]	**-0.42376**	(0.5037)	-0.84
	[**12**]	**0.02402**	(0.9240)	0.03
m	**Xbox**	**19135.29**	(1132.2)	16.90
	Xbox 360	**813.10**	(1884.7)	0.43
	PS2	**41135.91**	(848.8)	48.46
	PS3	**987.54**	(6416.3)	0.15
	GameCube	**16382.92**	(1194.1)	13.72
	Wii	**17385.63**	(2029.2)	8.57

Notes: m is in thousand units

Table 2.9: Multi-Generation Model for Video Game Consoles (Microsoft $\alpha = 1$, Sony $\alpha = 1$, Nintendo $\alpha = 1$)

Coefficient	Console	Estimate	s.e.	t-value
p	**Microsoft**	**0.01220**	(0.0043)	2.82
	Sony	**0.01142**	(0.0021)	5.49
	Nintendo	**0.00809**	(0.0016)	5.06
q	**Microsoft**	**0.05423**	(0.0201)	2.69
	Sony	**0.03216**	(0.0078)	4.13
	Nintendo	**0.06670**	(0.0151)	4.43
phis	[**1**]	**-0.09294**	(0.2166)	-0.43
	[**2**]	**-2.13156**	(1.5318)	-1.39
	[**3**]	**0.04654**	(0.2377)	0.20
	[**4**]	**-3.08154**	(1.6084)	-1.92
	[**5**]	**0.13089**	(0.1357)	0.96
	[**6**]	**0.40535**	(0.5606)	0.72
	[**7**]	**0.90676**	(0.2681)	3.38
	[**8**]	**0.37551**	(0.5191)	0.72
	[**9**]	**0.09675**	(0.2246)	0.43
	[**10**]	**0.43943**	(0.2651)	1.66
	[**11**]	**0.36342**	(0.7003)	0.52
	[**12**]	**0.15694**	(1.0118)	0.16
m	**Xbox**	**18908.97**	(1346.3)	14.05
	Xbox 360	**2685.35**	(874.9)	3.07
	PS2	**40409.41**	(1002.8)	40.30
	PS3	**2117.23**	(1142.5)	1.85
	GameCube	**17665.16**	(1570.8)	11.25
	Wii	**16224.15**	(2706.9)	5.99

Notes: m is in thousand units

Table 2.10: Multi-Generation Model for Video Game Consoles (Microsoft $\alpha = 0.3$, Sony $\alpha = 0.1$, Nintendo $\alpha = 1.1$)

	Xbox	**X360**	**PS2**	**PS3**	**GC**	**Wii**
Xbox	x	-1	[1]	[2]	[3]	[4]
X360	1	x	[5]	[6]	[7]	[8]
PS2	-[1]	-[5]	x	-1	[9]	[10]
PS3	-[2]	-[6]	1	x	[11]	[12]
GC	-[3]	-[7]	-[9]	-[11]	x	-1
Wii	-[4]	-[8]	-[10]	-[12]	1	x
Xbox	x	-1	-0.01	-0.07	0.02	**-2.39**
X360	1	x	**-0.29**	0.31	**0.62**	-0.20
PS2	0.01	**0.29**	x	-1	0.11	**0.60**
PS3	0.07	-0.31	1	x	-0.42	0.02
GC	-0.02	**-0.62**	-0.11	0.42	x	-1
Wii	**2.39**	0.20	**-0.60**	-0.02	1	x

Notes: the numbers between brackets represent the phi coefficients of the multi-generation model reported in table 2.9. The bold coefficients have t-values greater than 1.

Table 2.11: Competitive Parameters

Strategy	Player / Quantile	1%	5%	10%	50%	90%	95%	99%
[1] Random Selection of Dates	**Nintendo**	1681.4	6538.5	10933.5	26369.6	39146.8	42941.3	48084.3
	Sony	1550.5	2528.4	3890.3	10398.8	19962.5	23481.8	29284.3
	Total	13833.1	21187.1	26621.7	45590.7	57653.5	59676.6	62545.3
[2] Pre-commitment / Imitation	**Nintendo**	3769.5	9998.8	13288.1	28553.4	37470.9	39509.4	41273.4
	Sony	724.0	1431.3	2383.7	9415.3	18372.8	19513.3	20362.9
	Total	7324.8	15961.7	19174.6	38146.2	54316.4	57663.5	60415.2
[3] Pre-commitment / Optimization*	**Nintendo**	1543.9	4696.4	8540.6	28837.4	35805.5	36944.6	38842.0
	Sony	13890.7	15387.0	16973.4	21908.2	31948.0	33108.3	34149.6
	Total	17887.0	25253.3	30900.2	52132.8	60307.6	62785.7	65845.1
[5] Uncertain Launch Dates / Optimization**	**Nintendo**	1548.9	3834.1	6769.8	28615.2	36868.9	38215.9	39742.1
	Sony	13285.2	13873.2	14441.2	20579.3	30961.4	31846.6	32695.6
	Total	17335.7	24765.2	30434.0	48845.5	59968.6	62956.7	66081.8

Notes: * Nintendo pre-commits to a date while Sony optimizes its launch dates given Nintendo pre-commitment dates.** Nintendo does not pre-commits to any date and Sony optimize given the probability that Nintendo launch at any date.

Table 2.12: Evaluation of Four Launch Strategies

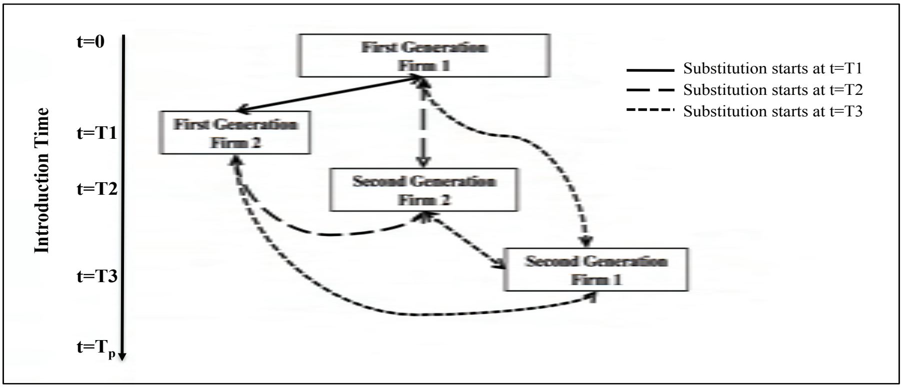

Figure 2.1: Interaction Between All Product Generations in Duopoly Model

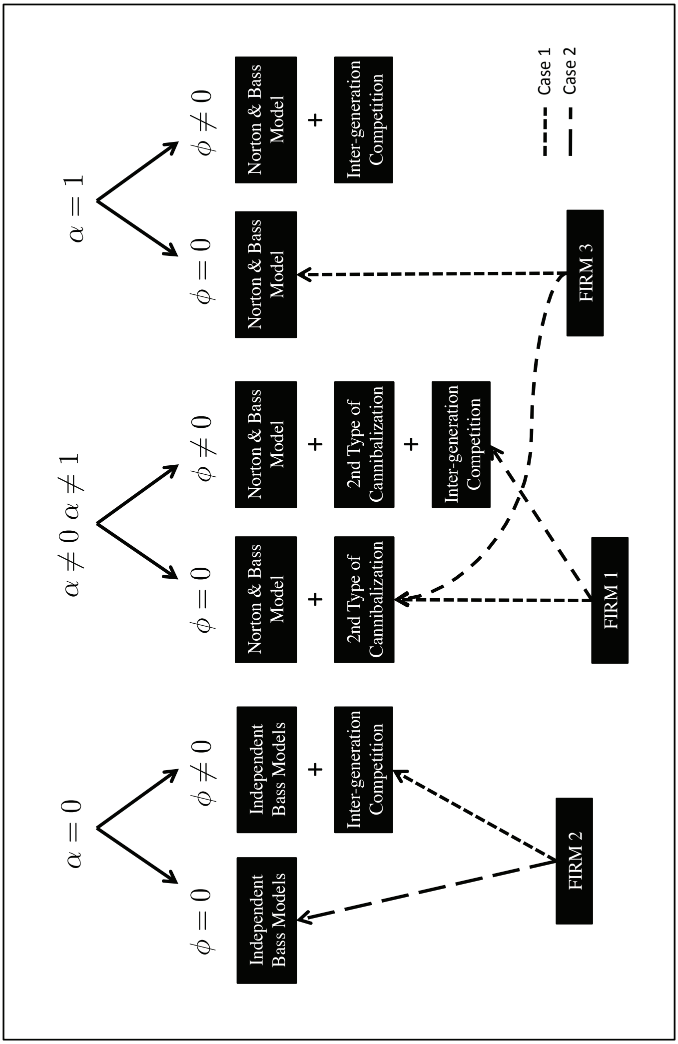

Figure 2.2: Model Relationship with Previous Research

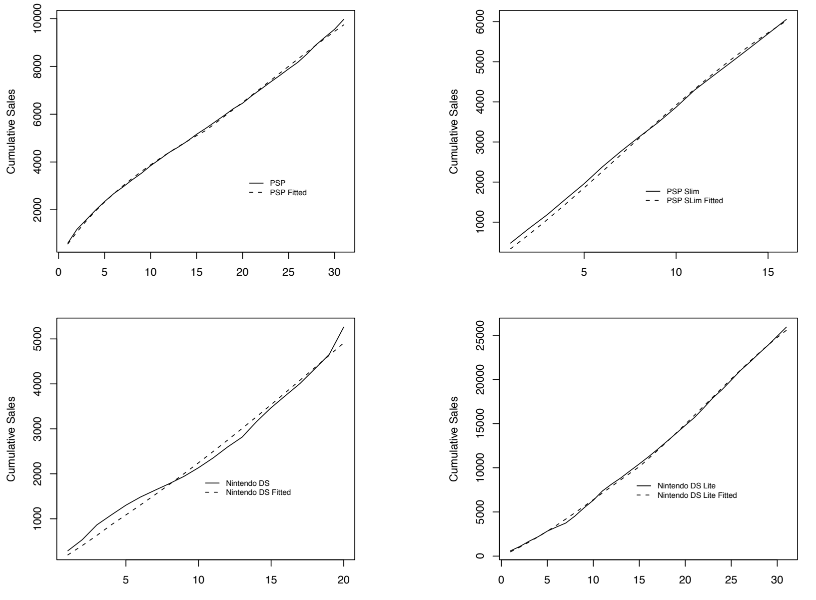

Figure 2.3: Multi-Generation Model Fit for Portable Systems

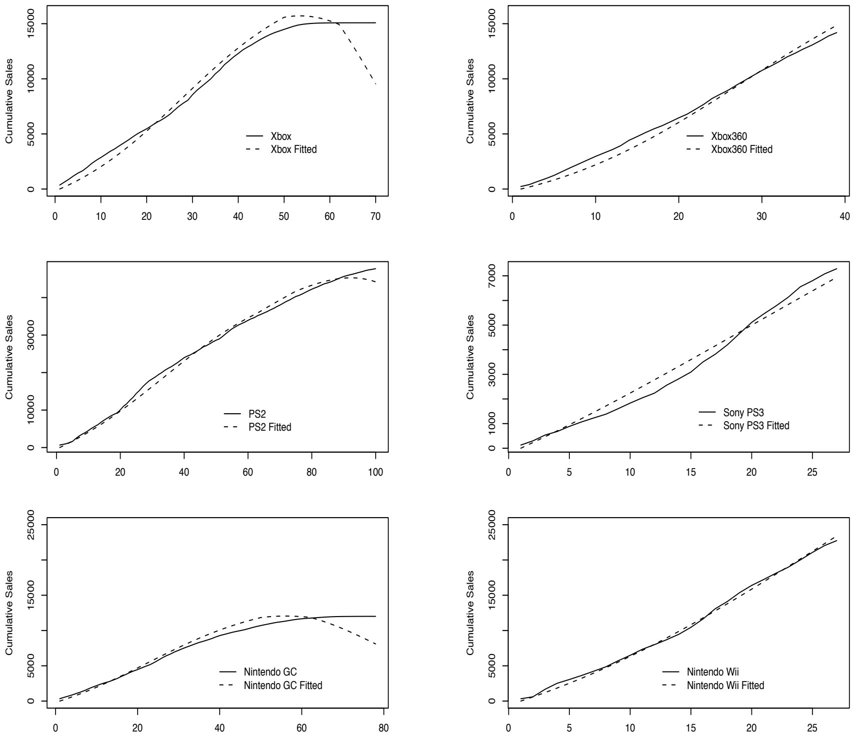

Figure 2.4: Multi-Generation Model Fit for Video Game Consoles

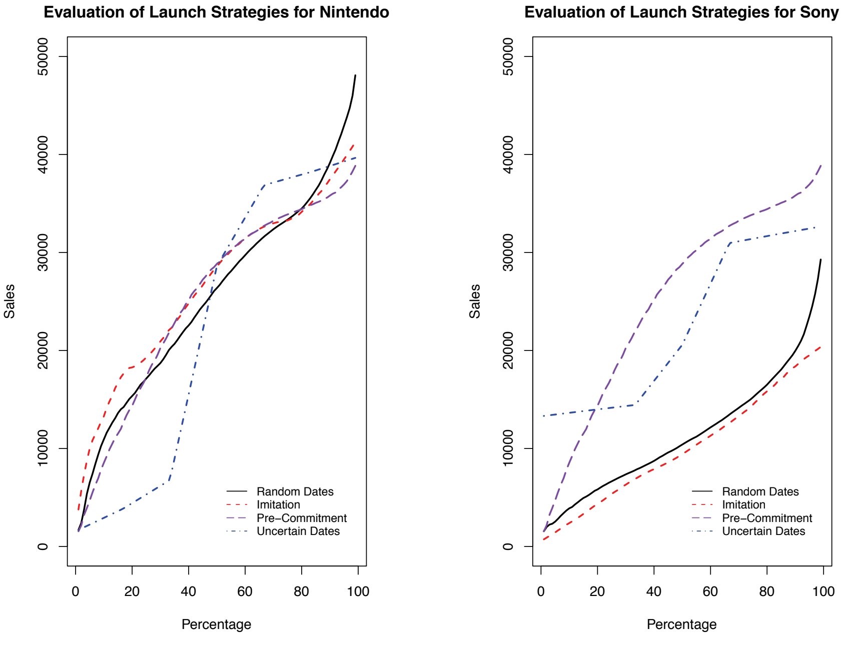

Figure 2.5: Cumulative Distribution Function of Sales given Different Strategies

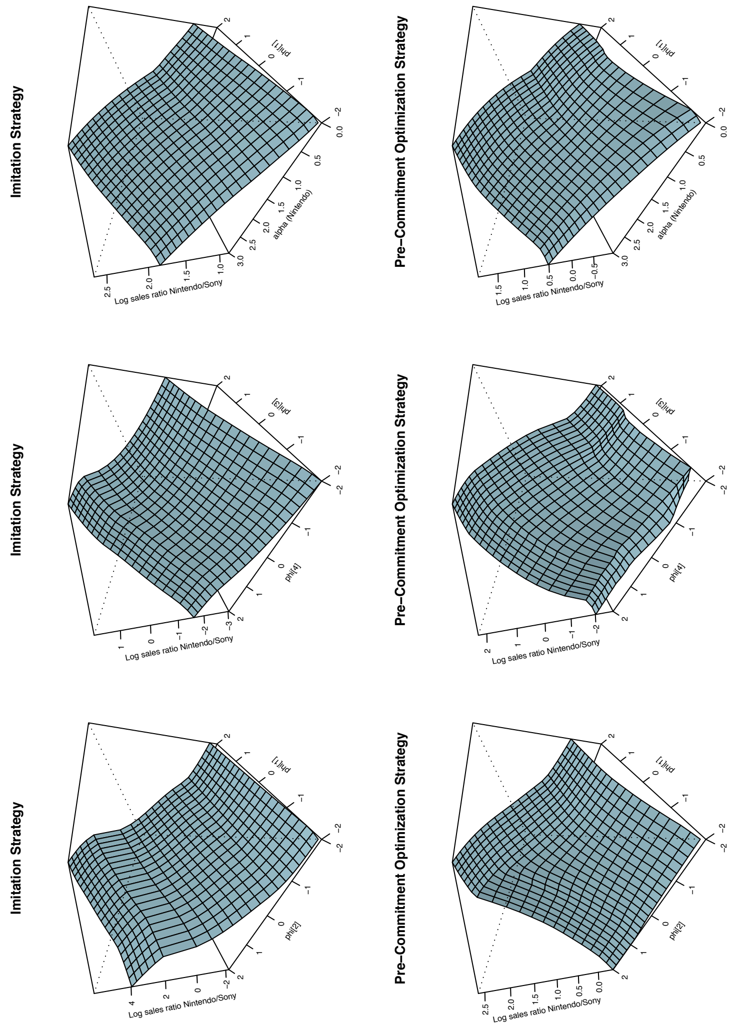

Figure 2.6: Strategy Sales Sensitivity to Competitive Parameters

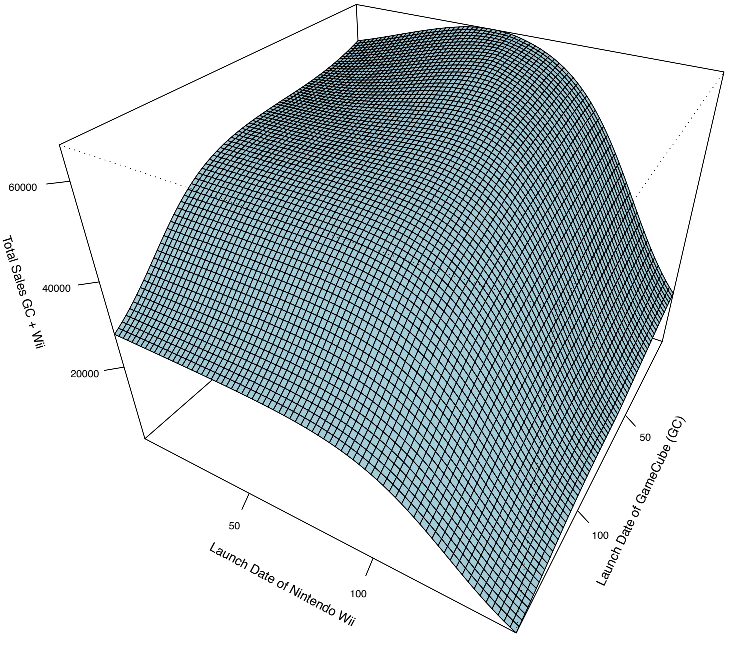

Figure 2.7: What if Scenarios for the Consoles of Nintendo

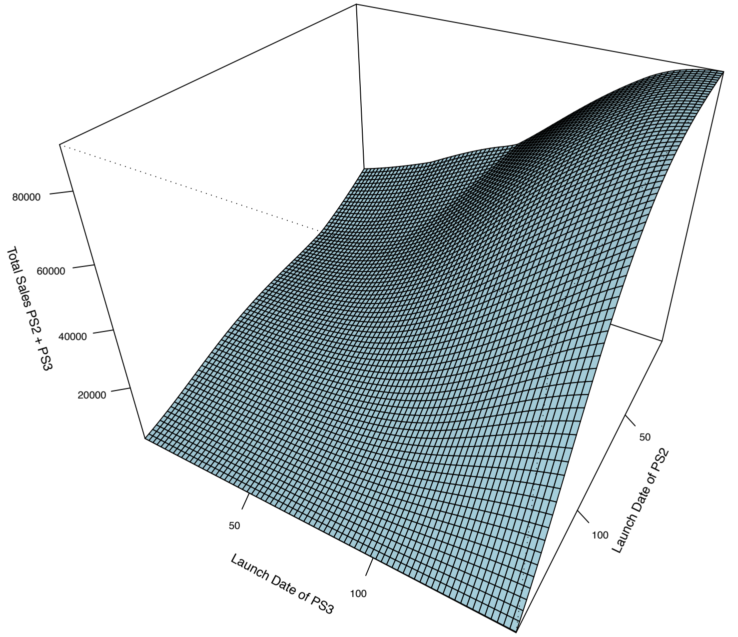

Figure 2.8: What If Scenarios for the Consoles of Sony

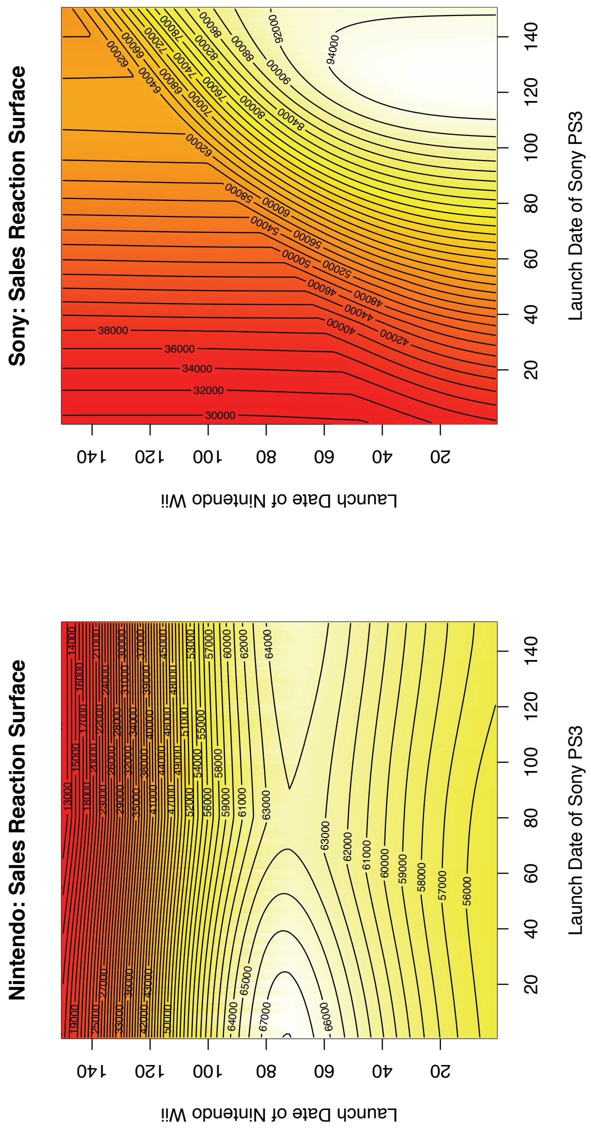

Figure 2.9: Sensitivity to Launch-Timing and Sales Reaction Surfaces

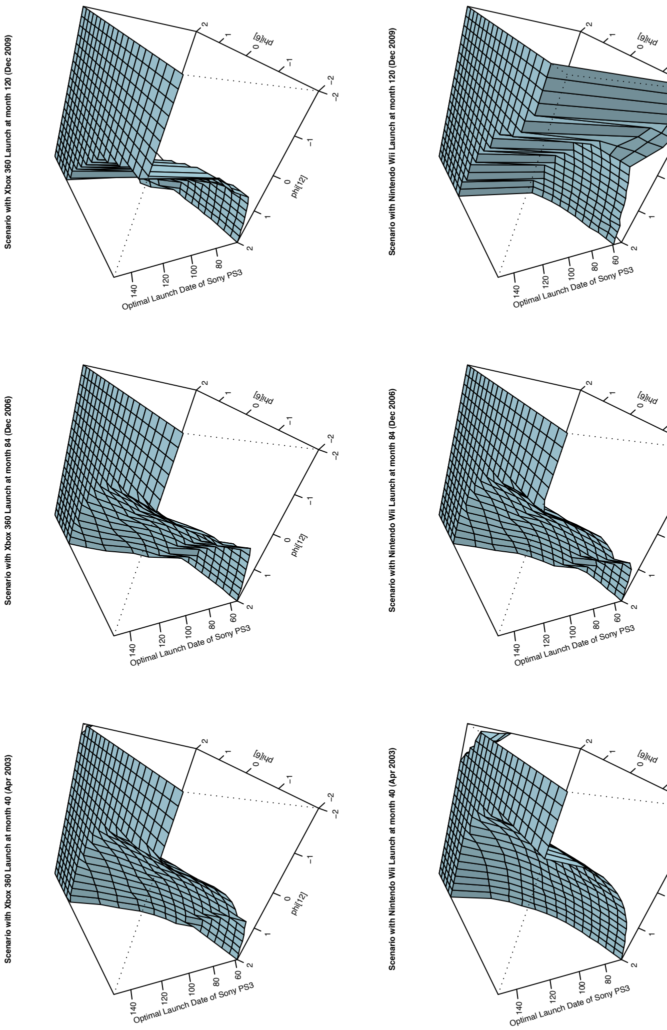

Figure 2.10: Sony PS3 Optimal Launch-Timing Sensitivity to Competitive Parameters

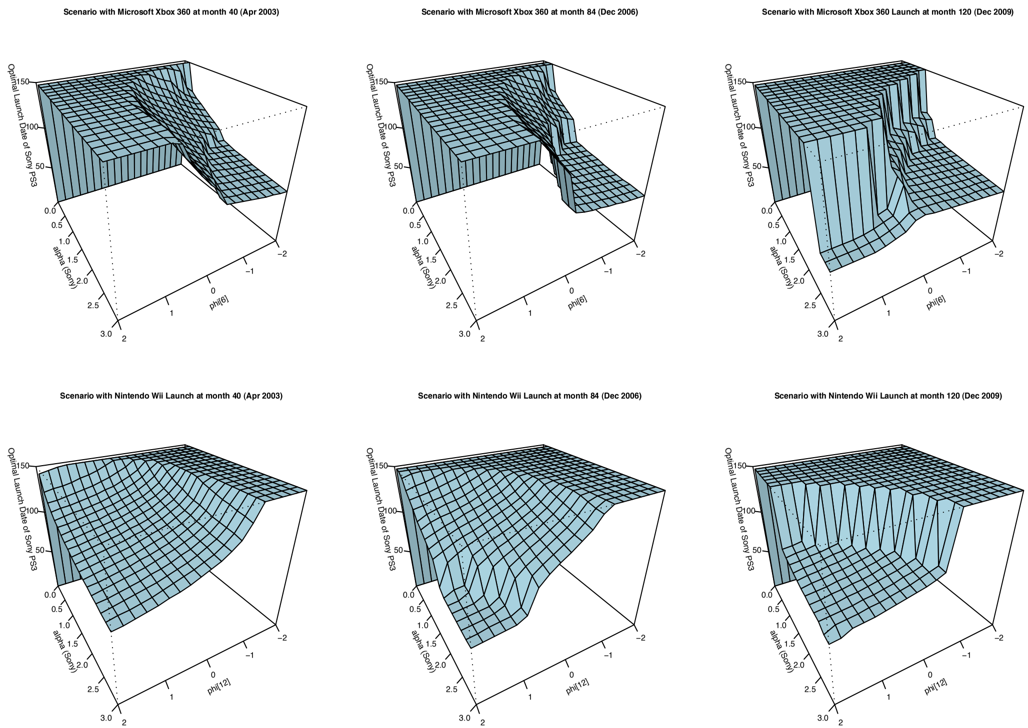

Figure 2.11: Sony PS3 Optimal Launch-Timing Sensitivity to Cannibalization and Competitive Parameters

2.A Strategy Simulation Methodology

In this section we provide the details of each of the four strategies we use in our application.

Random Date Selection

In this strategy firm 1 selects τ_1^1 and τ_2^1 randomly and firm 2 selects τ_1^2 and τ_2^2 in the same way. We discretize the planning horizon in T_p periods. Hence the possible launch dates τ_i^g (where i stands for firm i while g stands for the system generation) might be at any t within $t = 1, \ldots, T_p$; we denote the length of the planning horizon p. We consider a 90 month planning horizon that is 7.5 years. This time frame is long enough given the average life-cycle of the portable systems is 2.5 years. With this planning horizon we evaluate the maximum of the cumulative sales for each system of both firms given all the feasible launch dates τ_i^g for $i = 1, 2$ and $g = 1, 2$. That is, firm 1 might select one out of the p^2 possible launch-timings but we restrict the combinations to the set where $\tau_i^2 \geq \tau_i^1$. This means that we restrict that the second generation product for both firms is launched at a date either at the same time or after the first generation. The feasible set reduces from p^2 to $(p+1) \times p/2$ feasible combinations for each player. Note that we use $T_p = 90$ and we set $p = 45$. In the duopoly case $i = 1$ refers to Nintendo and $i = 2$ refers to Sony.

We evaluate equations (2.3) to (2.6) with the feasible set of launch-timing and we compute the maximum cumulative sales achieved by each product generation for both firms. That is we compute $\max(S_g^{Sony}(\tau_1^{Sony}, \tau_2^{Sony} | \tau_1^{Nin}, \tau_2^{Nin}))$ for $g = 1, 2$ and $\max(S_g^{Nin}(\tau_1^{Nin}, \tau_2^{Nin} | \tau_1^{Sony}, \tau_2^{Sony}))$ for $g = 1, 2$. The $g = 1$ product of Sony is the Sony PSP and the $g = 2$ product of Sony is PSP Slim; for Nintendo the $g = 1$ product is the DS and the $g = 2$ product is the DS Lite. In table 2.12 we report the quantiles of the total sales of Sony achieved by this strategy, that is $\sum_g \max(S_g^{Sony}(\tau_1^{Sony}, \tau_2^{Sony} | \tau_1^{Nin}, \tau_2^{Nin}))$ and $\sum_g \max(S_g^{Nin}(\tau_1^{Nin}, \tau_2^{Nin} | \tau_1^{Sony}, \tau_2^{Sony}))$ and the total sales of both players (the sum of the last two terms).

Imitation

In this strategy firm 1 pre-commits to a launch-timing for its two product generations while firm 2 imitates the launch-timing of firm 1. That is, both firms launch at the same time each of their product generations. In our application we set Nintendo the be the firm that pre-commits to a certain launch date and Sony to be the firm that imitates. We assume that Nintendo pre-commits to a randomly chosen pair of dates τ_1^{Nin} and τ_2^{Nin} and Sony sets $\tau_1^{Sony} = \tau_1^{Nin}$ and $\tau_2^{Sony} = \tau_2^{Nin}$. In this strategy we assume Nintendo ignores that Sony will imitate and we do not assume Nintendo might pre-commit strategically to the best pair of dates. However, it is straightforward to identify the best pre-commitment dates of Nintendo given Sony is imitating.

Pre-commitment and Optimization

In this strategy firm 1 pre-commits to a launch-timing for its two product generations while firm 2 optimizes its launch-timings given the launch dates of firm 1. As before, we set Nintendo the be the firm that pre-commits to a certain launch date and Sony to be the firm that optimizes. We assume that Nintendo pre-commits to a randomly chosen pair of dates τ_1^{Nin} and τ_2^{Nin} and Sony sets τ_1^{Sony} and τ_2^{Sony} such that $\sum_g \max(S_g^{Sony}(\tau_1^{Sony}, \tau_2^{Sony} | \tau_1^{Nin}, \tau_2^{Nin}))$ is maximized. In this strategy we assume Nintendo ignores that Sony will optimize and we do not assume Nintendo might pre-commit strategically to the best pair of dates. However, it is straightforward to identify the equilibrium if both firms are optimizing. Finally, we note that pre-commitment and perfect foresight are usual assumptions in the literature, for examples see Reinganum (1981) and Bayus et al. (1997).

Uncertain Launch Dates and Stochastic Optimization

In this strategy firm 1 selects a pair of launch dates for its two generation products but does not reveal these dates to firm 2. However, we assume firm 2 can derive the best response of firm 1 given any pair of dates assigned by firm 2 to its own products. That is, firm 2 has knowledge on the reaction function of firm 1 however firm 2 does not know which date launch will be picked for certain by firm 1. The reaction function is a function that maps any launch-timing of firm 2 to the best launch-timing of firm 1. In our application, the best reaction function of Nintendo

$$f(\tau_1^{Sony}, \tau_2^{Sony}) = \Omega\left(\max_{\tau_1^{Nin}, \tau_2^{Nin}} (\sum_g S_g^{Nin}(\tau_1^{Nin}, \tau_2^{Nin} | \tau_1^{Sony}, \tau_2^{Sony}))\right) \tag{2.23}$$

$\Omega()$ returns a pair of dates τ_1^{Nin} and τ_2^{Nin} that maximize the sales of Nintendo given the launch dates of Sony (τ_1^{Sony} and τ_2^{Sony}). We further assume that firm 2 assigns a probability that firm 1 will launch on dates τ_1^1 and τ_2^1 proportional to the sales achieved by selecting these two dates. That is,

$$p(\tau_1^{Nin}, \tau_2^{Nin}) = \frac{\sum_g \max(S_g^{Nin}(\tau_1^{Nin}, \tau_2^{Nin} | \tau_1^{Sony}, \tau_2^{Sony}))}{\sum_{(\tau_1^{Nin}, \tau_2^{Nin}) \in f(\tau_1^{Sony}, \tau_2^{Sony})} \sum_g \max(S_g^{Nin}(\tau_1^{Nin}, \tau_2^{Nin} | \tau_1^{Sony}, \tau_2^{Sony}))} \tag{2.24}$$

Note that τ_1^1 and τ_2^1 should belong to the set of dates given by the reaction function of firm 1 and that is why they should be contained in the reaction function $f(\tau_1^{Sony}, \tau_2^{Sony})$; otherwise the strategy is not considered. Given these assumptions the strategy of firm 2 is to select the pair of launch dates that maximize its expected sales. The best reaction function of Sony is

$$f(\tau_1^{Nin}, \tau_2^{Nin}) = \Omega\left(\max_{\tau_1^{Sony}, \tau_2^{Sony}} (\sum_g S_g^{Sony}(\tau_1^{Sony}, \tau_2^{Sony} | \tau_1^{Nin}, \tau_2^{Nin})) \right) \tag{2.25}$$

and hence Sony selects τ_1^{Sony} and τ_2^{Sony} such that

$$p(\tau_1^{Nin}, \tau_2^{Nin}) \times \sum_g \max(S_g^{Sony}(\tau_1^{Sony}, \tau_2^{Sony} | \tau_1^{Nin}, \tau_2^{Nin}))_{(\tau_1^{Sony}, \tau_2^{Sony}) \in f(\tau_1^{Nin}, \tau_2^{Nin})} \tag{2.26}$$

is maximized.

Chapter 3

The Timing and Speed of New Product Price Landings

Many high-tech products and durable goods exhibit exactly one significant price cut some time after their launch. We call this sudden transition from high to low prices the *price landing*. In this chapter we present a new model that describes two important features of price landings: their timing and their speed.

Prior literature suggests that prices might be driven by sales, product line pricing, competitor's sales or simply by time. We propose a model using mixture components that identifies which of these explanations is the most likely *trigger* of price landings. We define *triggers* as thresholds after which prices are significantly cut. In addition, price landings might differ across products and therefore we model their heterogeneity with a hierarchical structure that depends mainly on firm, product type and seasonal effects.

We estimate our model parameters applying Bayesian methodology and we use a rich dataset containing the sales and prices of 1195 newly released video games (VG's). In contrast with previous literature, we find that competition and time itself are the main triggers of price landings while past sales and product line are less likely triggers. Moreover, we find substantial heterogeneity in the timing and speed of price landing across firms and product types.

3.1 Introduction

"*Don't get us wrong – price cuts are a good thing*"

Wired.com (2007)

It is well known that prices of new products exhibit one or several important price cuts during their life-cycle. Nowadays, we are witnessing how many new high-technology products are introduced at an initial high price and after a certain moment their prices are cut to a permanent and much lower level. This practice is commonly followed by manufacturers of products like video games, apparel, PCs, movies, and so on. Moreover, scholars have recognized and studied this type of pricing strategy. For example, studies like Feng and Gallego (1995) and Gupta et al. (2006) point that managers at apparel retailers in New York City report the timing and depth of price cuts are important decision variables and the depth of the price cut in this industry is typically between 25 and 50%. In this chapter, we will call this sudden transition from an initially high price to a lower price level the *price landing*.

We are not aware of any empirical study of price landings. This is quite a surprise because the timing of a permanent price cut for a new product is without a doubt an important managerial decision. During the first half of 2007 thousands of American customers purchased Apple's iPhone and they witnessed a \$200 price drop just 66 days after its release. Consumers were outraged by the sudden price drop and Apple apologized and issued a \$100 store credit to everyone who purchased the iPhone before April 2007. More recently, the forthcoming market launch of Apple's iPad has brought attention to the pricing strategy that the Apple Store will apply to e-books. According to journalists, Apple is pushing the industry to apply "variable pricing which apparently is *triggered* by sales volume and not just pricing whim", see Wired Magazine (2010). In some instances Apple's timing of price cuts have been judged too early if they happened short time before the Christmas season and in other instances the price cuts have been judged as occurring too late to stimulate further sales or to fight competition. See BusinessWeek Online (2007) and BusinessWeek Online (2008) for more details on Apple's story.

In this chapter we present a new model for *price landings* and the estimation approach we present is particularly useful to describe the moment and speed at which the *price landing* occurs and to simultaneously find the *triggers* of these sudden price transitions. Our work offers a complement to studies like those of Tellis et al. (2003) and Golder and Tellis (1997) because we characterize and describe pricing patters of new products while these latter authors have studied and characterized new products' sales patterns. On the other hand, our modeling approach goes further than a description of price patterns because it allows us to find what are the most likely *triggers* of price landings. We apply our model to the market of video games and to a rich data set that concerns 1195 newly released video games.

The plan of the chapter is as follows. In Section 3.2 we present our literature review. In Section 3.3 we present our data and market context. Next in Section 3.4 we present our modeling approach and in Section 3.5 we present our results. We present our conclusions in Section 3.6. All figures and tables are presented in Appendix 3.7. We present the estimation approach in Appendix 3.A.

3.2 Literature Review

In this section we review the studies concerned with the video-game industry in subsection 3.2.1 and next in subsection 3.2.2 we review the literature related to new products pricing.

3.2.1 Research on Video-Games

Three empirical studies closely related to our work are Clements and Ohashi (2005), Nair (2007) and Chintagunta et al. (2009).

Clements and Ohashi (2005) study the indirect network effects between video game consoles and video games and the effects of consoles' prices on their own sales. Their findings suggest that price elasticity is low at the beginning and high at the end of the life cycle of video game consoles. Chintagunta et al. (2009) investigate the effects of software availability and prices on the sales of video game consoles. They propose an econometric approach that accounts for the endogeneity of price and sales and they find time varying price elasticities. In contrast with Clements and Ohashi (2005), Chintagunta et al. (2009) find some evidence of both declining and increasing elasticities. Other studies, like Parker (1992) and Simon (1979), report that elasticities may show diverse time profiles across products, like U or inverted U shapes. See Parker (1992, Table 4, page 365).

Nair (2007) studies the video-game software market and he proposes a model that takes into account the interaction between publishers of video-games and two consumer segments formed by high and low valuation gamers. His findings suggest that the optimal pricing by publishers should exhibit declining prices. The price cut rate (that is the slope of the price function) in Nair (2007) depends on the relative size of each of the consumer segments while the overall and initial level of the optimal price depends on the utility discounting factor and the interaction of consumers and firms.

Our study differs markedly from Clements and Ohashi (2005), Nair (2007) and Chintagunta et al. (2009) because our objective is to introduce a model that is flexible enough to capture many different and detailed theoretical features of prices that have been documented in the literature or observed empirically. In this respect, our price model is a generalization based on previous research. In addition, we offer the first empirical study that focuses on price landings and their triggers, timing and speed.

Finally, the methods of Clements and Ohashi (2005), Chintagunta et al. (2009) and Nair (2007) are considered structural while our model may be classified as a reduced form model. A main advantage of our reduced form is that we do not need assumptions regarding supply and demand side interactions or consumer behavior. A disadvantage of our approach is that we can not draw inferences regarding consumer behavior or consumer-firm interactions and that we need assumptions on the form of the price equation. However, the assumptions we will use for the price equation are more flexible than the assumptions of Nair (2007) and Chintagunta et al. (2009). Nair assumes that consumers form expectations based on an auto-regressive process of order one while Chintagunta and colleagues assume that prices are stationary. In contrast, we present a very flexible equation that can capture sudden breaks (non-stationarity) and it allows us investigate what is *triggering* these breaks. Hence, we offer novel findings and we are the first to measure quantitatively empirical features of prices that have not been documented before in the literature. In addition, our econometric approach is computationally simple. Therefore, we can use our method to study relatively large databases of prices. This may be a technical advantage over structural models that are usually much more computationally demanding.

3.2.2 Research on New Products Pricing

The literature dealing with pricing strategy is extensive and in this section we focus our attention to a set of empirical and analytical studies concerned mainly with new product prices. We present the studies we surveyed in Table 3.1.

In Table 3.1, we see that 24 out of 32 studies are analytical while 8 are empirical. Out of these eight empirical studies only Clements and Ohashi (2005), Chintagunta et al. (2009) and Nair (2007) were published recently and only the study of Nair (2007) is focused on pricing policies for new products. To our knowledge, Nair (2007) and our work are the only empirical studies concerned with price patterns. A likely reason of such lack of empirical studies on prices is the scarcity of detailed price data.

We draw the following generalizations the literature in Table 3.1: 1. Prices show gradual or sudden transition from high to low states. Both empirical and theoretical studies have documented such transitions. 2. Prices show transitions that rarely mimic the S-shape of sales or that increase over time (8 studies). 3. Prices respond to competition, changes in consumer valuations across time, consumer heterogeneity, new product releases, learning curves on costs and market saturation.

The first generalization tell us that prices of new products rarely stay constant. We note that some studies, like Schmalen (1982), Ferguson and Koenigsberg (2007) and Eliashberg and Jeuland (1986), have shown when it is optimal to keep prices of new products constant. On the other hand, we could hardly draw a consensus about how fast price transitions should be or how they look empirically. Some studies explicitly report the optimal price decrease rate, like in Dockner and Gaunersdorfer (1996), Raman and Chatterjee (1995) and Bayus (1994) while many other studies give less attention specifically to the speed of price transitions. Much more is known about the shape of price transitions. Many studies, like Robinson and Lakhani (1975), Kalish (1983), Dolan and Jeuland (1981), Bayus (1992), show that the optimal policy is for prices to decline over time. Other studies show the optimal mark-down (or optimal sudden price discount) based on the length of the season, the perishability of the product or drastic seasonal changes in consumer valuations or demand. See for example Ferguson and Koenigsberg (2007), Gupta et al. (2006), Rajan et al. (1992) and Feng and Gallego (1995). Finally, diffusion studies, like Rao and Bass (1985), confirm that the declining pattern is an empirical regularity and recent studies, following Bass et al. (1994), usually incorporate the declining price effect on diffusion.

The literature suggests the generalization that prices should change (in most cases drop) once an event modifies the market and that these price drops occur in synchrony with the movements of price drivers. These events are usually related to the drivers listed in the last column of Table 3.1. In general terms, previous *empirical* literature suggests that x *drives* y when x is an important underlying variable causing the variance in y. In contrast, many *analytical* studies integrate *trigger* variables into their models where x is defined as a *trigger* of y if it has an effect on y only after a certain threshold, for example after $x > x_o$ becomes true where $x > x_o$ might mean, for example, competitive entry, the end of a season or the limit of market potential. For example, Feng and Gallego (1995) and Gupta et al. (2006) incorporate thresholds after which prices should be marked down. We believe there is a disconnect between analytical studies that allow non-linearities and sudden price breaks and empirical studies that assume in most cases linear price functions without structural breaks.

The objective of this chapter is to fill the literature gaps between empirical and analytical studies of new products' prices. First, our model, together with the econometric approach we use, will allow us describe the theoretical features of prices based on a large database of prices. We focus specifically on the speed and timing of sudden price transitions, what we call *price landings*. Second, we test the relative importance of different price triggers suggested by theoretical and empirical studies simultaneously. We test whether saturation, market entry, time (a products' age) or the release schedule of firms trigger the price landing for each of the 1195 products in our data set. In this way, we put to an empirical test the theoretical properties of prices discussed in analytical studies and we connect both streams of research.

3.3 Video Game Prices

In this section we first describe our data and next we present a brief description of the video game market.

3.3.1 Data

The database we analyze consists of monthly time series of unit sales and prices for 1195 PlayStation2 (PS2) video games released between September 1995 and February 2002 in the US. This data was collected by NPD Group from retailers that account for 65% of the US market. We used the first two years of data for each video game and left out VG's with less than 12 monthly observations. This time frame is justified by the fact that most VG's stay on store shelves for less than two years and their sales drop very rapidly to zero afterwards. Binken and Stremersch (2009) use the same data and they assume that a video game is in a so-called *dead regime* after its sales drop below 5000 units. Therefore, Binken and Stremersch (2009) do not use any observation after this cut-off point which leaves out 32 % of their observations. In our case the 24 month cut-off point leaves out 38 % of the observations. We compared our results against a 30 and a 36 month cut-off point that leave out 28 and 20 % of the observations, respectively, and our results are qualitatively the same. Our final sample consists of 1075 video games.

In Figure 3.1 we show the price landing of 50 randomly selected video games. This figure clearly shows the great diversity of price patterns but it is easy to see of the common feature across games: their price drops at a certain moment in time. The introductory prices range from 40 to approximately 60 USD while their landing level is between 15 and 30 USD. Similarly, there is great diversity in the timing of price landings. It is easy to notice that some VG's prices drop right after the second month while others land around the 10th, 12th or 15th month or even later. Finally we notice that some prices drop very fast, see the lines almost parallel to the vertical axis, while in many other cases they land at slower rates and with more noise around them.

In Figure 3.2 we show the price landing of one of the most popular VG's, the Spider-Man game. We plot the price of the Spider-Man game on the vertical axis but in each of the panels we use a different scale on the horizontal axis. In the upper-left panel we use time on the horizontal axis, in the upper right panel we use the cumulative sales of Spider-Man and in the lower panel we use the cumulative number of VG's launched to the market after the introduction of Spider-Man. We choose these axes because later we will identify each of these variables as a potential trigger of price landings. More details on this are given in Section 3.4.3. These graphs of course show very similar price patterns. That is, we could say that the price cut of the Spider-Man occurred approximately at the 10th month after its introduction (upper-left panel); or just after reaching 600 thousand unit sales (upper-right panel); or after 250 VG's were launched (lower panel). The price landings in these figures are similar but the interpretation of the different thresholds is very different. In all cases, these thresholds represent an event after which prices drop, that is the timing of price landings. Finally, if we look closely at the different price landing patterns we discover that the speed of landing varies across these panels. Prices seem to drop much faster when we use cumulative sales than when we use time on the horizontal axis.

In the analysis that follows we show how we select one of these potential price landing triggers for each of the products in our sample. Specifically, in Section 3.4.3 we present how we use our mixture specification and the underlying distributions of price landings to select among potentially correlated price landing triggers. Developing a joint model for prices, sales and competitive entry is beyond the scope of this chapter and we consider it as an area for future research. We explain more details on our modeling approach in Section 3.4 and in this section we continue with a presentation of the market context of our application.

3.3.2 The Video Game Market

The video game market is highly competitive and there are 78 video game publishers who design games for PS2. On average, they released 29 new video games per month between 1992 and 2005. The main publisher of these VG's is Sony and it has a market share of 16%. Acclaim and Electronic Arts follow Sony with market shares of 11% and 6%, respectively. In the upper left panel of Figure 3.3 we present the distribution of the market shares across all publishers. We notice that 20 publishers have about 80% of the market while the 58 remaining publishers cover the next 20% of the market. In the upper right panel of Figure 3.3 we depict the monthly time series of the number of newly released video games. There is an upward trend in the number of VG's being released. In 1996 less than 11 VG's were released per month while in 2002 this volume has increased to 40 monthly releases.

The bottom left panel of Figure 3.3 shows the industry's sales pattern. Total VG's sales are extremely seasonal and they peak every December when they may reach numbers like 14 million copies. This last number is especially high if we compare it against the 24.1 million units of PS2 consoles sold between 1995 and 2002. Finally, in the lower right panel we show the average number of video games released from 1995 to 2002 and the average sales per month. An interesting fact is that most new VG's are released during November and January but sales peak in between these two months. From 1995 to 2002, December VG's unit sales are on average 14 million and in January sales decrease to less than 3 million copies while on average 18 new VG's are released on December, 27 in November and 34 in January. In Figure 3.4 we can see the distribution of the type of video games sold. For example, sports games account for 21.5 %, Action 14 % while Strategy games account for 4 % of all VG's in our data.

The consumers in this market concern 40 million US-based consumers who buy video games each year. Figure 3.5 shows a histogram of the total sales across all VG's. Preferences clearly differ across VG's as we observe substantial heterogeneity in the market potential across the video games. We follow the tradition of diffusion research by labeling the cumulative sales reached by a video game as the market potential. From Figure 3.5 we can learn that sales above one million units for a single game seem to occur only rarely. The average market potential for the video games in our sample is around 254.75 thousand units. However, approximately for half of the VG's in our sample (to be precise: for 504 video games) the market potential is less than 66 thousand units.

3.4 Price Landing: Modeling

In this section we present our modeling approach. The model and econometric approach we present allow us measure quantitatively the theoretical features of prices discussed in our literature review. Specifically, the equation we propose allows us describe the speed and timing of price transitions while we use mixture modeling to test the relative importance of different price landing triggers. Finally, we apply a hierarchical structure to describe the empirical distributions of the timing and speed of price transitions and at the same time to identify the most likely price triggers.

Our model consists of two parts. First we present an equation to describe the price landing, that is the underlying price of product i at time t, which we call $P_i^*(t)$. Next we specify an equation that relates the pricing landing to the actually observed prices, what we call $P_i(t)$. As we observe in Figure 3.1, prices follow a general inverse S-shape but they do not follow it very smoothly and in most cases the prices we observe are noisy. Hence, in the first equation we capture the price landing and its main two features (timing and speed) and in the second we capture deviations from it. In Section 3.4.1 we present these two equations. Each video-game is allowed to have its own price landing speed, timing, initial price and landing price parameters. In Section 3.4.2 we therefore specify how we model this heterogeneity. In Section 3.4.3 we briefly discuss the mixture specification that allows us to identify the trigger of the price landing for each video game. In Section 3.4.4 we discuss heterogeneity in mixture probabilities. In Section 3.4.5 we present details regarding the co-variates in the hierarchical structure of the model.

3.4.1 Price Landing Model

The price landing of game i is $P_i^*(t)$ and we assume it depends on a trigger denoted by $D_i(t)$. That is, prices change according to

$$\frac{dP_i^*(t)}{dD_i(t)} = \frac{(P_i^*(t) - \kappa_i)(\rho_i - P_i^*(t))}{(\kappa_i - \rho_i)\nu_i}, \tag{3.1}$$

where ρ_i is the starting price level, κ_i is the final pricing level, and ν_i a constant that moderates the rate of change $dP_i^*(t)/dD_i(t)$. For ease of interpretation, $D_i(t)$ might be for example time and then $dP_i^*(t)/dD_i(t) = dP_i^*(t)/dt$. $D_i(t)$ can be set to be any trigger variable that we are interested in, like sales or competition. From (3.1) we see that a smaller ν_i implies a faster rate of change. Here, the time index t will in each case be relative to the launch date of the particular product. In other words for each product $t = 0$ corresponds to the time of launch. In the numerator of (1) we have that the closer $P_i^*(t)$ is to its initial or final levels, the slower prices would change and that if $P_i^*(t) < \rho_i$, $\nu_i > 0$, $P_i^*(t) > \kappa_i$, $\rho_i > \kappa_i$ for all t then $dP_i^*(t)/dD_i(t) < 0$. These last conditions describe very closely the price patterns that are common among high-tech products.

Equation (3.1) may be unusual in the sense that it models dP/dD instead of dD/dP. However, in our application we will use different trigger variables for D and hence dD/dP would not have the common interpretation we find in the literature when D are sales; for example, D could be competitive introductions. The former is the typical solution proposed by analytical studies while the latter is the typical form assumed in empirical studies. One of the possible reasons why empirical studies have assumed this latter form is that many of them focus on a single firm, usually a monopolist that sets prices. In contrast, in our study we observe the $\frac{dP_i^*(t)}{dD_i(t)}$ for hundredths of products launched by 78 firms that are price setters. Hence, our objective is to characterize the heterogeneity of $\frac{dP_i^*(t)}{dD_i(t)}$ across products and to capture two of its features, the timing (λ_i) and speed (ν_i) of significant price cuts. In addition, the advantage of equation (3.1) is that we can solve it analytically and test it empirically. In fact, it can be shown that (3.1) is a separable differential equation and that its solution is

$$P_i^*(t) = \kappa_i + (\rho_i - \kappa_i)h_i(t), \tag{3.2}$$

with

$$h_i(t) = 1 - \frac{e^{\left(\frac{D_i(t)-\lambda_i}{\nu_i}\right)}}{1 + e^{\left(\frac{D_i(t)-\lambda_i}{\nu_i}\right)}}. \tag{3.3}$$

That is, we propose that the price of product i is composed of two parts, a fixed landing price (κ_i) plus a mark-up ($\rho_i - \kappa_i$) that evolves over time proportionally to $h_i(t)$. The function $h_i(t)$ gives the percentage of the markup at time t and it is bounded between 0 and 1. The function (3.3) for $h_i(t)$ follows a logistic shape and λ_i can be interpreted as the location of the price landing for product i in terms of the trigger $D_i(t)$ while ν_i is the speed at which the landing occurs. That is, we observe a price drop after $D_i(t)$ reaches its threshold λ_i and this is why we call $D_i(t)$ the *trigger variable*.

The advantage of a logistic function for the pricing equation is that we can interpret its parameters in a natural way in our application. We plot equation (3.1) for $D_i(t) = t$ and different values of λ_i and ν_i in Figure 3.6. As can be noticed from the graph, the effect of an increase (decrease) of λ_i is to shift the complete function to the right (left) and ν_i has the role of smoothing the function or steepening the function. That is, ν_i is a parameter that determines how fast prices are falling and λ_i captures the moment (event) when prices are dropping.

In principle, $D_i(t)$ can be any variable that increases monotonously. The simplest choice for $D_i(t)$ is simply time ($D_i(t) = t$). It is important to notice that the interpretation of λ_i and ν_i depend on the choice of $D_i(t)$. If we set $D_i(t)$ to be the cumulative sales of product i then λ_i is simply the number of items sold at high prices. We might interpret this limit as a proxy for the size of the segment that buys at high prices; what some call the *hard-core gamer* segment. This is a natural interpretation for λ_i but we do not claim that this model really identifies who and how many are the real *hard-core gamers*. Furthermore, if we define $D_i(t)$ as the number of products introduced after launch of product i then λ_i becomes a competitive threshold after which prices are cut. In all cases ν_i is a scaling constant that marks the transition speed of prices as we set in equation (3.1) and it of course depends on the scale of $D_i(t)$. Notice that $D_i(t)$ might be a combination of different trigger variables. The interpretation of the λ_i parameters then becomes troublesome with such specification.

As discussed above, $P_i^*(t)$ aims to capture the underlying price pattern of product i, that we call price landing. For actual data we observe this pattern plus noise. The observed prices may therefore differ from $P_i^*(t)$. Furthermore, we only observe the prices at regularly spaced intervals. We adopt the convention that we observe the prices for product i at $t = 0, 1, 2, \ldots, T_i$. We denote the observed price at time t by $P_i(t)$. We model the relation between the observed prices and price landing pattern using a first order auto-regressive specification. In terms of the observed price this gives

$$P_i(t) = P_i^*(t) + \alpha_i[P_i(t-1) - P_i^*(t-1)] + \varepsilon_i(t) \qquad t = 1, 2, 3, \ldots, T_i, \tag{3.4}$$

where $\varepsilon_i(t)$ denotes the source of the random deviation at time t from the underlying price landing pattern, and α_i determines the memory in the deviations from the underlying pattern. We assume that $\varepsilon_i(t) \sim N(0, \sigma_i^2)$ for $t = 0, 1, \ldots, T_i$. If $\alpha_i = 0$ there is no memory, and (3.4) then states that the deviations are independent over time. If $\alpha_i > 0$, a positive deviation at time t is likely to induce a positive deviation at time $t + 1$. For the first observation we set

$$P_i(0) = P_i^*(0) + \sqrt{\frac{1}{1 - \alpha^2}} \times \varepsilon_i(0). \tag{3.5}$$

The variance factor is set such that the variance of the random term equals the unconditional variance of $P_i(t)$ in (3.4).

3.4.2 Heterogeneity in Main Parameters

In the above discussion of the model we have explicitly allowed for heterogeneity, that is, all parameters and the price cut trigger $D_i(t)$ are product-specific. In this section we discuss how we model the heterogeneity in all parameters.

In the model we will allow for K different triggers, which are denoted by $D_{1i}(t)$, $D_{2i}(t)$, ..., $D_{Ki}(t)$. The relationship between the observed price and the price landing in (3.2) remains unchanged. In addition, we define a different price landing equation $P_{ki}^*(t)$ for each trigger variable k, that is,

$$\begin{aligned} P_{ki}^*(t) &= \kappa_i + (\rho_i - \kappa_i) h_{ki}(t) \\ h_{ki}(t) &= 1 - \frac{e^{\left(\frac{D_{ki}(t) - \lambda_{ki}}{\nu_{ki}}\right)}}{1 + e^{\left(\frac{D_{ki}(t) - \lambda_{ki}}{\nu_{ki}}\right)}}. \end{aligned} \tag{3.6}$$

Note that this definition is very similar to that in (3.2) and (3.3). However, the parameters λ_{ki} and ν_{ki} are now trigger (k) and product (i) specific. Note that the price starting and landing level ρ_i and κ_i are the same across all k possible triggers.

The landing level (κ_i), the initial price level (ρ_i), the threshold value (λ_{ki}) and the speed of adjustment (ν_{ki}) are defined to vary across products. For each of these parameters we specify a second-level model. For the price landing level and the launch prices we specify

$$\left.\begin{aligned} \kappa_i &= Z_i'\gamma^\kappa + \omega_i^\kappa \\ \rho_i &= Z_i'\gamma^\rho + \omega_i^\rho \end{aligned}\right\} \text{ with } (\omega_i^\kappa, \omega_i^\rho) \sim N(0, \Sigma), \tag{3.7}$$

where Z_i denotes a vector of dimension M of product specific characteristics, γ^κ and γ^ρ are coefficient vectors (dimension $M \times 1$) common across all i products. The error terms ω_i^κ and ω_i^ρ are assumed to be normal with mean 0 and covariance matrix Σ. The Z_i in our model will include mainly product type, manufacturer variables and seasonal dummies. We define the Z_i variables with more detail in Section 3.4.5. We specify a similar form for the speed and timing parameters. That is, for each trigger variable k we define

$$\left.\begin{aligned} \ln \lambda_{ki} &= Z_i'\gamma_k^\lambda + \eta_{ki}^\lambda \\ \ln \nu_{ki} &= Z_i'\gamma_k^\nu + \eta_{ki}^\nu \end{aligned}\right\} \text{ with } (\eta_{ki}^\lambda, \eta_{ki}^\nu)' \sim N(0, \Omega_k). \tag{3.8}$$

where η_{ki}^λ and η_{ki}^ν are the error terms and they are assumed to be normal with mean 0 and covariance matrix Ω_k. The γ_k^λ and γ_k^ν are coefficients vectors (dimension M) and Z_i are the same group of group of covariates as in the equations for κ_i and ρ_i. The log transformation in (3.8) is used to ensure that λ_{ki} and ν_{ki} are positive. If it is the case that the timing and the speed of price landings are correlated we will capture this correlation with the matrix Ω_k. For example, it might be that when prices fall at a slower rate (ν_i^k) they are cut at an earlier time (λ_i^k).

3.4.3 Choice of Trigger and Mixture Specification

The actual trigger of the price landing for each product is of course unobserved to the *researcher*. We denote this (unobserved) variable as S_i, that is, we denote $S_i = k$ if the trigger variable k is selected for product i. We complete this part of the model by specifying probabilities for each trigger, that is, the trigger k is selected with probability π_k for $k = 1, 2, \ldots, K$. In our application $k = 1$ would mean that *time* is the trigger, $k = 2$ means that *cumulative sales* are the trigger and $k = 3$ means that *cumulative competitive introductions* are the main trigger of equation (3.2). In the four-trigger version of our model $k = 4$ means that the release schedule of firms are the main trigger. We provide more details on how we measure each trigger variable in subsection 3.4.5. The probabilities π_k will reflect the overall likelihood of each of the different triggers. However, note that conditionally on the observed prices, the probability of $S_i = k$ is different across games.

In Figure 3.7 we describe the intuition about how triggers are selected and statistically identified. For this purpose we need two main elements. The first element consists of the distributions of the threshold parameters for each of the different triggers. That is, the distribution of λ_{ki} and ν_{ki} across all i and for each k. For example, if we collect the parameter λ_{1i} for all i we obtain the distribution of λ for the first trigger variable. As we defined in equation (3.8), the distribution of λ_{ki} and ν_{ki} depend on co-variates Z_i and hyper-parameters γ_k and the variance term associated to them. The second element we need is the match between the price landing of game i and the distributions of λ_{ki} and ν_{ki} for $k = 1, \ldots, K$.

In Figure 3.7 we plot again the price of the Spider-Man. In addition, we plot a hypothetical distribution of the threshold parameters λ_{ki} for each of the mixture components k. The distribution of λ_{1i} in the upper left panel, λ_{2i} in the upper right panel and λ_{3i} in the lower left panel. Note that λ_{1i} is the time (in months) after which the price drops (if $D_i(t)$ is time, that is when $k = 1$). In the same way, if $D_i(t)$ is cumulative sales then λ_{2i} is the cumulative number of sales after which the price drops and λ_{3i} is the cumulative number of competitive introductions after which the price drops when $D_i(t)$ amounts to competitive introductions. We notice that the $\hat{\lambda}_{1i} \approx 11$ months, that $\hat{\lambda}_{2i} \approx 600$ thousand units and that $\hat{\lambda}_{3i} \approx 250$ units. Given the λ_{ki} thresholds we can now compare them against the corresponding distributions. In this case we see that the $\hat{\lambda}_{2i}$ is the closest to the mode of its corresponding distribution. Hence, the most likely trigger of the Spider-Man price landing is sales. The least likely trigger is competition and next is time. Of course, in our model we take into account the distribution of λ_{ki} and ν_{ki} simultaneously when we draw the most likely trigger for each video game in our sample. All technical details about trigger selection are given in the Appendix 3.A. Next we describe how we model heterogeneity in the mixture components.

3.4.4 Heterogeneity in Mixture Probabilities

We suspect that there also might be heterogeneity in the mixture probabilities across games. For example, the games of some publishers may be more likely to belong to the time mixture. Hence, as an extension to the model we allow the probabilities of $S_i = k$ to depend on a set of product specific variables. To model this dependence we specify a Multinomial Probit Model for S_i. Hence, we introduce additional latent variables y_i^* for $i = 1, \ldots, N$ and $k = 1, \ldots, K$. These latent variables are related to S_i by

$$S_i = k \quad \text{if and only if} \quad y_{ki}^* = \max_{l=1\ldots K}(y_{li}^*). \tag{3.9}$$

We specify y_{ki}^* as

$$y_{ki}^* = Z_i'\delta_k + \vartheta_{ik} \quad \text{with} \quad \vartheta_i \sim N(0, I), \tag{3.10}$$

where $\vartheta_{\mathbf{i}} = (\vartheta_{1i,}\vartheta_{2i,}, ..., \vartheta_{Ki})$ and we set $\delta_1 = 0$ for identification. In principle the set of variables used in this specification may differ from that in (6) and (7). The probability that the trigger k is used for product i now becomes

$$\pi_{ki} = \Pr[y_{ki}^* = \max_{l=1\ldots K}(y_{li}^*)]. \tag{3.11}$$

This concludes our model specification. For inference we will rely on MCMC and Bayesian analysis and treat all product specific parameters as latent variables and we sample these together with the parameters in (3.6), (3.7) and (3.8). A complete description of the sampling steps in this Markov Chain can be found in the Appendix 3.A.

3.4.5 Model Specifics for Video Games Pricing Model

We consider two versions of our model. The first version uses three trigger variables and the second uses four trigger variables. We define $D_{ki}(t)$, for $k = 1, 2, 3, 4$ where $D_{1i}(t) = A_i(t)$,

$D_{2i}(t) = C_i(t)$ and $D_{3i}(t) = I_i(t)$ and $D_{4i}(t) = R_i(t)$. $A_i(t)$ is defined as the age of a video game in months, that is, the time between launch and t. $C_i(t)$ is the cumulative sales of video game i between release date and t. $I_i(t)$ is defined as the cumulative number of video games introduced between the launch date of video game i and t. $R_i(t)$ is defined as the release schedule of the firm that released product i. We know the number of games a firm released at every point in time. To create $R_i(t)$, we use a time window that sums the introductions from the introduction of game i up to the next three months after t.

The interpretation of λ_{ki} and ν_{ki} varies depending on the trigger k. Hence, λ_{1i} can be interpreted as the price landing time, λ_{2i} as a competitive threshold, λ_{3i} as the hard-core gamer segment size and λ_{4i} as a release limit after which we observe a price drop. For each of these triggers, the parameter ν_{ki} for $k = 1, 2, 3, 4$ can be interpreted as a scaling constant that changes the speed at which the price landing occurs.

In all what follows in this section we focus on the model with three triggers, that is $k = 1, 2, 3$ and we leave out $R_i(t)$. The reason for this is that $R_i(t)$ is selected with a probability very close to zero when we include it as the fourth trigger variable. We present the discussion regarding the fourth trigger in our results in section 3.5.2.

The hierarchical structure of the corresponding threshold λ_{ki}, speed ν_{ki} and ρ_i and κ_i parameters for each mixture component will depend on a set of Z_i variables that contain game type, publisher and seasonal effects plus the launch price and the time to the introduction of a new game consoles as co-variates. Seasonal dummies are defined by the month of launch of each video game i. The launch price is the observed price of video game i at launch time, that is at $t = 0$. We include this variable in order to test if our co-variates remain significant after including past prices in the equation for the timing and speed of launch. It might be that the price at launch of a VG might contain information regarding the timing of the price landing and its speed. In addition, we believe it is reasonable to include the launch price because of the very likely uni-directional relationship between launch price and timing of price landing. That is, it is very hard to argue that a firm decides how to price a VG's based on its decision on when to permanently cut its price; on the other hand, it might be that firms decide to cut prices based on the launch price. For example, firms might cut the price of expensive games after longer time than the time they wait to cut the price of cheaper VG's. Moreover, the launch price is a proxy for quality and hence we test if our covariates remain significant after we control for them.

The time to console launch measures the time (in months) between a video game release and the launch of the VG's console that is being released after the video game introduction. For example, the PlayStation2 with DS controllers was introduced in June 1998 and other versions of the PS2 console were released in February 1999 and January 2002. This means that a video game released in January 1998 will face a console introduction after 6 months; a video game released in January 1997 will face a release in 18 months, and so on. For each video game we calculate the time between its release and the forthcoming console at the video game release date. We include this variable to test whether the price landing pattern varies relative to the release date of video game consoles. Our results do not significantly change if we leave both time to console launch and launch price out of the Z_i covariates.

From the seasonal fixed effects we excluded January, from the game types we excluded Adventure games. The remaining game type categories are: Action, Arcade, Children, Driving, Family, Fighting, Role playing, Shooter, Sports, Strategy and Compilations. The remaining publisher dummies are Electronic Arts, Acclaim, Infogames, Konami, Activision, Midway, Eidos Interactive, THQ, Capcom, Namco, Agetec, Interplay, Hasbro, 2$^{\text{nd}}$ group, 3$^{\text{rd}}$ group and 4$^{\text{th}}$ group. The 2$^{\text{nd}}$ group is composed by six publishers that each have at least 1% market share, the 3$^{\text{nd}}$ group is composed by 14 publishers that account for the next 10% market share and the 4$^{\text{th}}$ group is composed by 43 publishers that account for less than 1% of the market share in total. In all our tables we sorted publishers by their market share and in descending order. The main publishers (EA, Acclaim, etc.) account for 80% of the VG's in our sample while the dummies for 2$^{\text{nd}}$, 3$^{\text{rd}}$ and 4$^{\text{th}}$ publishers group the next 20% of the market share. We set Sony as the reference publisher.

3.5 Results

In this section we present our results in three subsections. In the first we present results regarding the heterogeneity of the parameters. Next we present the results regarding trigger selection and finally we discuss the model performance.

3.5.1 Heterogeneity of Landing Time and Speed

Our results indicate that there indeed exists heterogeneity in the model parameters. The first contribution we have to offer is that we find significant firm effects on both the timing and speed parameters across all mixtures. That is, firms might be deciding not only on when to cut the price but also on how fast to cut it. To our knowledge, this result is new and we are the first to show it empirically. In Table 3.2 and in Table 3.3 we can see the different firm effects across mixtures and model parameters. For example, Acclaim's landing time (λ_i) coefficient in the time mixture is -0.196 and this means that VG's of Acclaim face a price drop 17.8% earlier relative to Sony. In addition, we find several of the firm effects on the landing speed (ν_i) to be significant. For example, Electronic Arts has a ν that is 91.7 % higher than Sony while Agetec has a slower landing speed with a ν parameter that is 3.81 times higher than Sony. An interesting feature of the time mixture parameters is that most of the firm effects $log(\lambda_{1i})$ are negative while the firm effects for the $log(\nu_{1i})$ are positive. That is, it seems that the video game prices of most firms are cut at earlier dates than Sony but most firms cut prices at slower speed relative to Sony.

In the last four columns of Table 3.3 we report the results for the hierarchical specification of the initial and landing price levels, (3.7). In both cases we observe very important firm effects. For example, Konami sets the landing prices 2.535 USD above the landing prices of Sony, 17.34 USD, while the launch prices of Konami are not significantly different than those of Sony that start at 40.49.

We give a histogram of the posterior mean of the game-specific parameters of the three-mixture model in Figure 3.8, Figure 3.9 and for the auto-regressive term of equation (3.4) in Figure 3.10. The dispersion in the timing and speed parameters is reported in Figure 3.9. We can see that each mixture has quite different thresholds and speeds. For example, the time mixture mean is around 7 months. That is, firms cut VG's prices mainly in the 7th month after their release. The timing parameters for all mixtures are graphed in the left frames while in the right frames we present the speed parameter distribution. We note that if the speed parameter ν_i is close to zero then prices fall more steeply. From the histograms in the right panels of Figure 3.9 we see that several products face sharp price cuts. In addition, in Figure 3.8 we see the distribution of the starting price level ρ_i for all i in the left frame and the distribution of the κ_i in the right frame. These parameters show that the starting level might be as low as 20 USD and as high as 70 USD while the landing level is as low as 5 USD and as high as 35 USD.

In summary we find that firm effects are important to describe the price landing timing and to describe its speed, the launch and the landing prices of the VG's in our sample. Seasonality is more important for the starting and landing levels of prices and less so for the price landing timing and speed. We also find that for some mixtures the effect of the launch price and the time to launch a new console are significant for some of the main parameters.

3.5.2 Triggers of Price Landings

Our second contribution is that we find that the triggers that best describe price landings are competitive introductions and time and not cumulative sales. In Figure 3.11 we report a histogram of the posterior probability of each of the triggers across all games in the three-mixture version of our model.

The academic convention is that sales should be an important price driver. In contrast, we find that the sales indicator is the least likely trigger variable of price landings and it is useful to explain only a 12% of the video-games in our sample. Note that we do not go against the academic convention that posits that sales are a price *driver*. Our results only indicate that sales are not the main price landing *trigger*. Furthermore, we find that the competition indicator, measured by competitive VG's introductions, is the likely trigger of price landings of approximately 25.7% of the VGs in our sample. The study of Nair (2007) finds no evidence of important substitution patterns between video-games and hence he suggests that competition, at the game-specific level, is not important to explain video-game prices. Our model cannot provide insights regarding the individual level competition between different video-games but we find that competition, measured as the cummulative sum of VG's introductions, is a likely trigger of price landings. Finally, we find that the most likely trigger is time itself or in other words, the most probable trigger is simply the *age* of a video-game. The time mixture has a posterior mean probability of 62.21%.

In the fourth-mixture version of our model we tested a fourth trigger without much success. The additional mixture included the release schedule of firms as trigger. The idea was to test whether firms release schedule could have a large probability relative to the other three trigger variables. Firms usually have information on the dates that their new VG's are to be released and therefore the prices of their previously released VG's could depend on the dates of these new introductions. Our data include the number of games each firm released at every point in time and therefore we also know the number of games each firm will release after each point in time. Hence, we sum the VG's introductions before time t up to the introductions in the next three months after t and this sum is the value of $D_i(t)$. Note that $D_i(t)$ is then the release schedule of the manufacturer of the video game i at time t. We decided to use a three-months time window because most online sources of VG's releases cover, as a maximum, the upcoming three months. Of course, in our database we just know the release schedule perfectly. However, our results indicate that the probability of this latter trigger mixture is on average very close zero. Our conclusion is that price landings are better described by the entire market introductions rather than the release schedule of any single firm. This makes some sense given that the 78 VG's firms in our sample face on average 29 releases per month. Consequently, firms might be more likely to monitor all market introductions rather than their own product introductions.

3.5.3 Model with Hierarchical Specification in the Mixture Probabilities

We estimate the same specification of our model but now we add a hierarchical specification in the mixture probabilities. In this section we discuss the estimates of this hierarchical specification and in the Appendix 3.A we provide its technical details.

The estimates of the parameters in the hierarchical structure of the mixture probabilities are reported in Table 3.4. In contrast with the heterogeneity in the main parameters we do not find substantial heterogeneity in the mixture probabilities. For example, we find only three significant publisher effects (Konami, Activision, Midway) in the latent utility of sales and two significant publisher effects (Capcom and Interplay) in the latent utility of the time mixture. That is, we know that there is heterogeneity in the timing and speed of price landings but we do not know why a trigger is more likely than the others. We consider this an area for further research.

3.5.4 Model Performance

We compare the out-of-sample performance of our model against two models: A naive model for prices, that is an AR(1), and against an alternative version of our model. In this alternative model we use the same specification and parameters and the same number of mixtures as our model but we replace all triggers with time. That is, $D_i(t) = time$ for all k mixture components. We randomly selected 50 video-games and used their first six observations to forecast their complete series. That is, only the first six observations of these 50 games were used for parameter estimation while we continue to use all observations for all other games. These comparisons are reported in Table 3.5 and in Table 3.6.

Our model preforms extremely well when compared against the AR(1) model and reasonably well when compared against the restricted model. In Table 3.5 we see that our model forecasts prices better than a naive AR(1) model for 40 out of the 50 randomly chosen games. We report the root mean square forecast error and the log of the predictive density for all 50 VG's. More details on how we compute the predictive density are given in the Appendix 3.A. Moreover, our model performs better than the model with three time mixtures for 19 out of the same 50 games and in 18 other cases it performs equally well as the alternative specification. In total 37 out of 50 games our specification performs at least as well as the alternative or better. This means that there is information contained in past sales and the competition mixtures that increase our model fit.

The AR(1) model does not capture the timing of significant price cuts and the speed at which the price cut occurs while our model captures these significant price cuts. Nonetheless, the assumption that prices follow an AR(1) pattern is common in previous marketing literature and our evidence suggests that this model performs poorly. The main reason is that new products face significant price cuts during their life-cycle and hence the AR(1) is not a suitable specification for such price patterns. At the moment and to our knowledge, we are the first to propose an empirical model that captures these price dynamics.

The results we presented in the previous sections are robust to different model specifications. For example, we estimated the model without the hierarchical specifications of all its parameters and the price landing timing and speed parameters stay about the same. Furthermore, we estimated the model without the auto-regressive structure in (3.4) and (3.5), and again the main parameters are estimated similarly. A reason why our results stay the same is that the pricing equation in (3.2) can accommodate many different pricing patterns with only four parameters and that we let these parameters to be product-specific. These four parameters are the initial and landing price levels, ρ_i and κ_i, and the timing and speed of price landings, the λ_i and ν_i.

3.6 Conclusions

Our aim with this chapter was to model the dynamics of new product price landing patterns. Price landings usually follow the inverse of the well known S-shape of sales. Nonetheless, we found no empirical studies dealing with these regularities of new product prices.

In this chapter we were concerned with products that face one significant price cut during their life cycle. Several online price trackers report similar dynamics to a wider range of products like mobile phones, cameras, storage media, books, etc. Our data was collected by NPD Group but several websites like www.pricescan.com or www.streetprices.com let their users plot price trends and indeed it is relatively easy to find many other products facing a single and significant price drop during their lifetime. We believe that knowing when a price is cut or when to significantly cut the price of a product permanently is an exciting area of further research and one with wide managerial implications across different industries.

In this chapter we provided evidence that there is heterogeneity both in the timing and speed of price landings. We found that most of this heterogeneity is driven by firm effects. Our model captures this heterogeneity and it is flexible and useful to forecast and describe the price landing patterns in our data. Finally, we found that it is the age of a video game that is triggering the price landings. The next most likely trigger is competition and the least likely is cumulative sales. This latter finding goes against the academic convention that sales are the main *driver* of prices. At least for our application we found convincing evidence that sales are not the most likely *trigger* of significant price cuts.

3.7 Figures and Tables

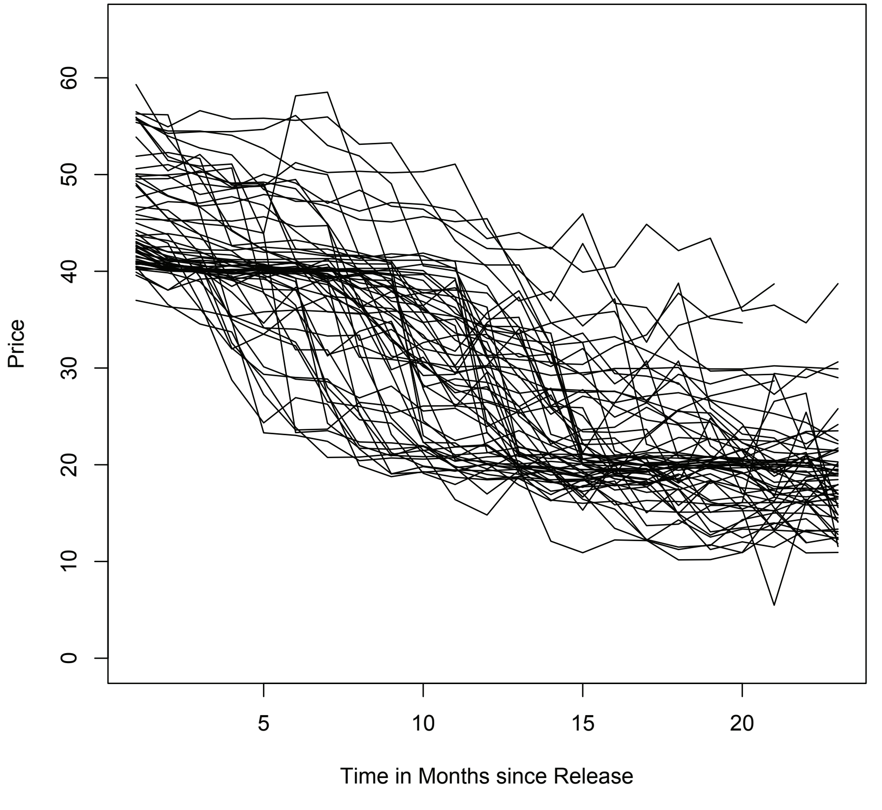

Figure 3.1: Price Landing Pattern for 50 Randomly Selected Games

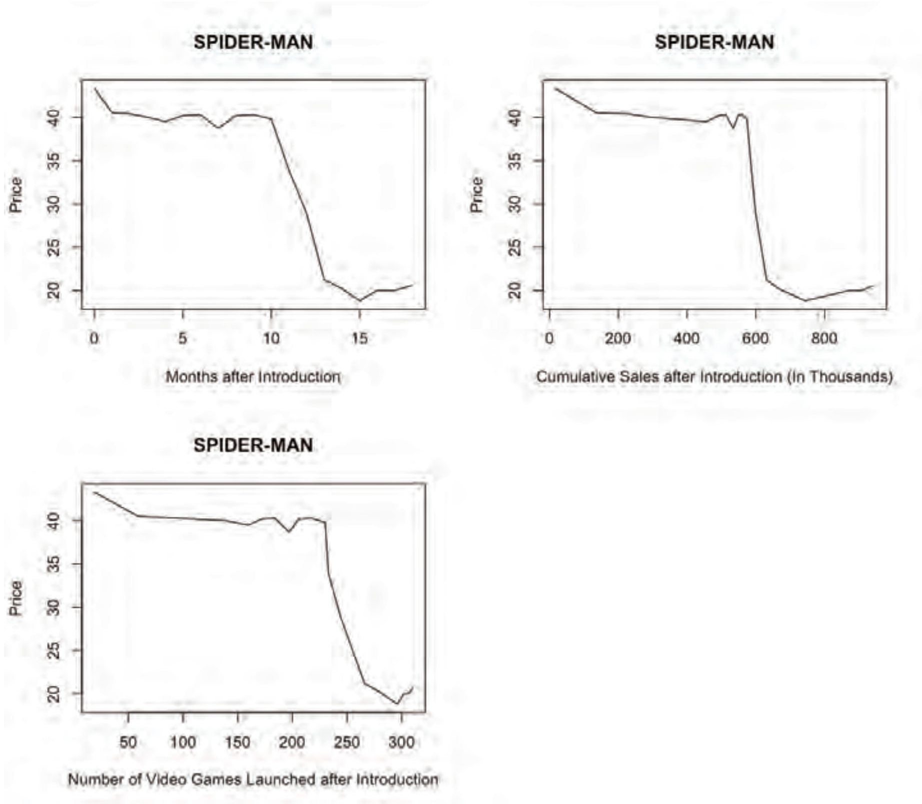

Figure 3.2: Typical Price Landing Pattern

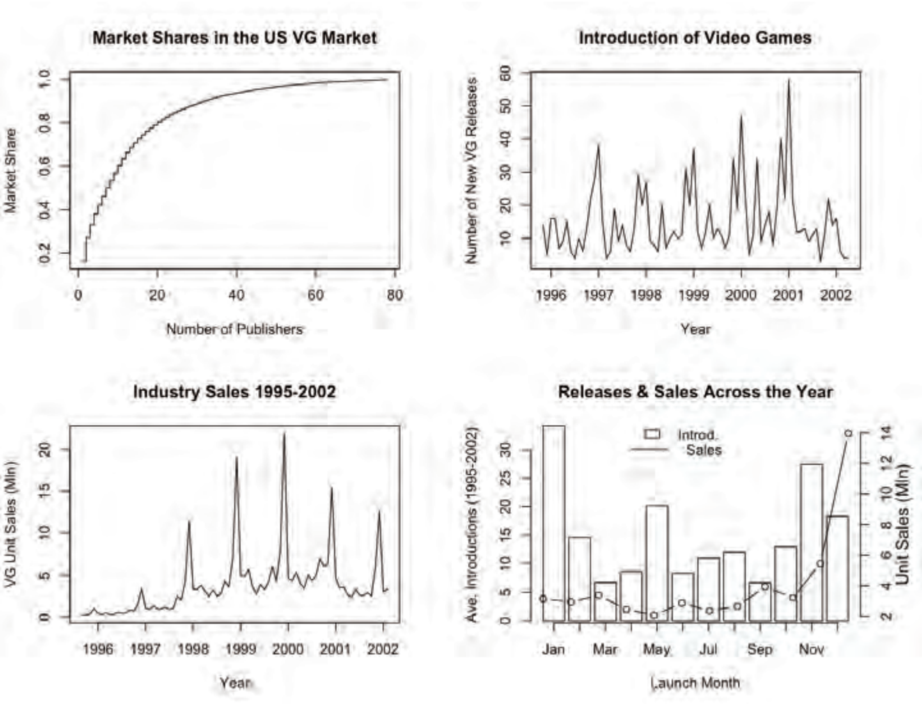

Figure 3.3: The Video Games Market

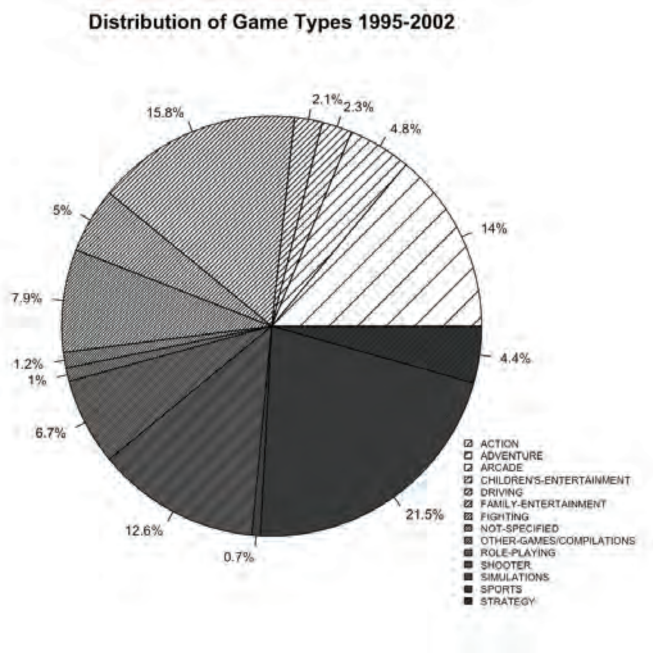

Figure 3.4: What do publishers sell?

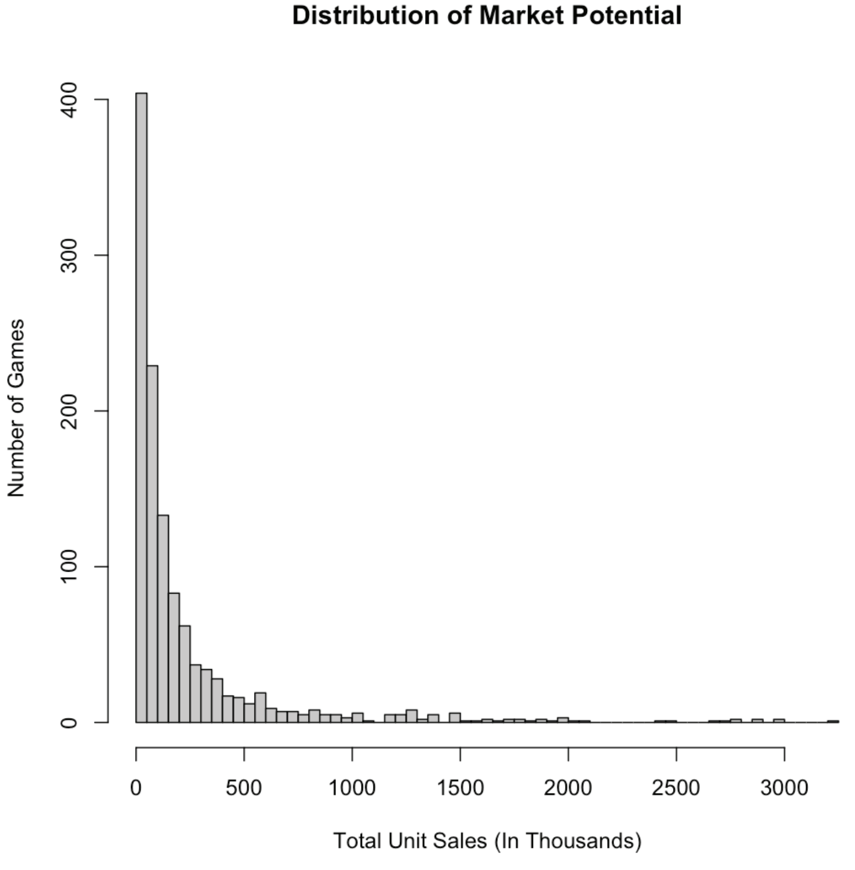

Figure 3.5: Total Sales Distribution

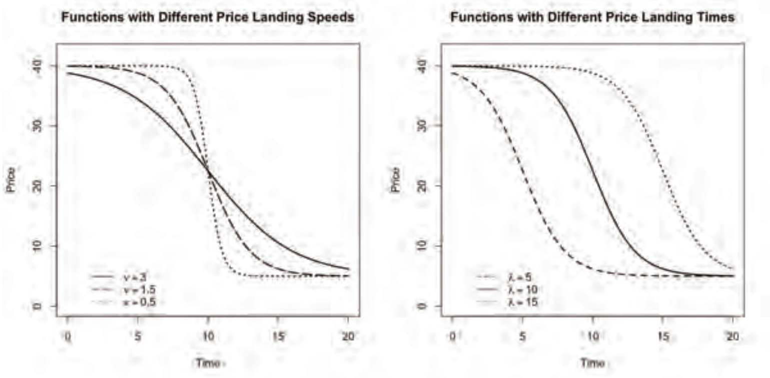

Figure 3.6: Main Pricing Function at Different Parameter Values

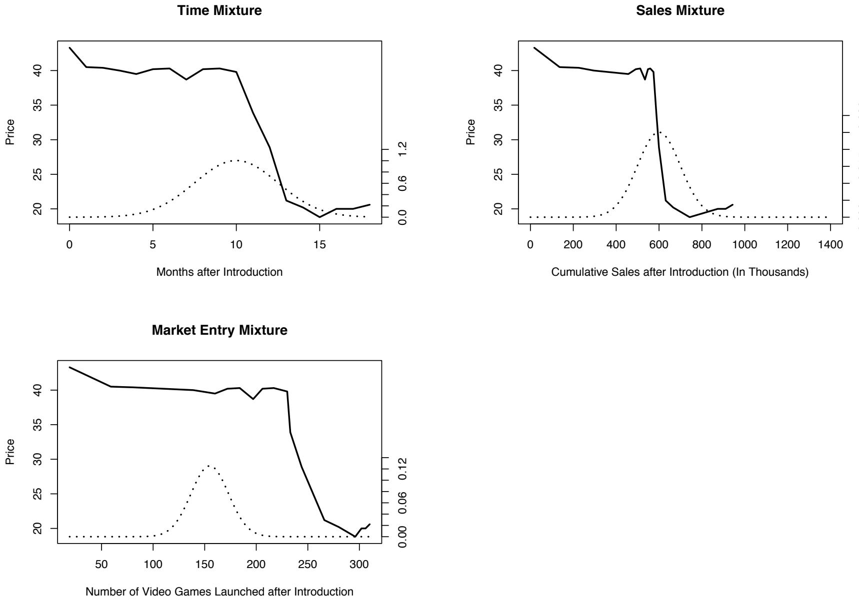

Figure 3.7: Identification of Triggers

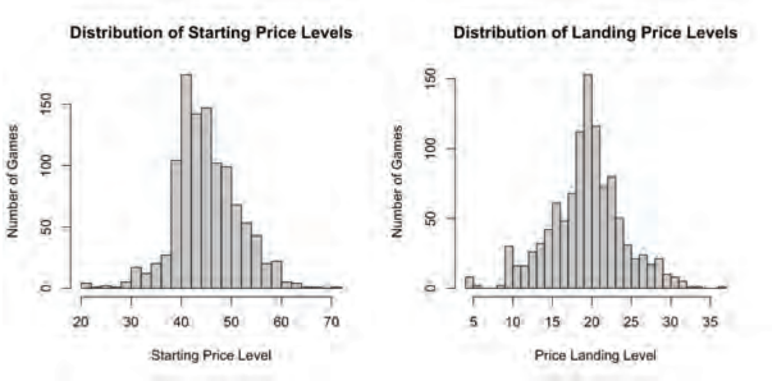

Figure 3.8: Histogram of the Posterior Mean of Starting (ρ_i) and Landing Price (κ_i) Parameters

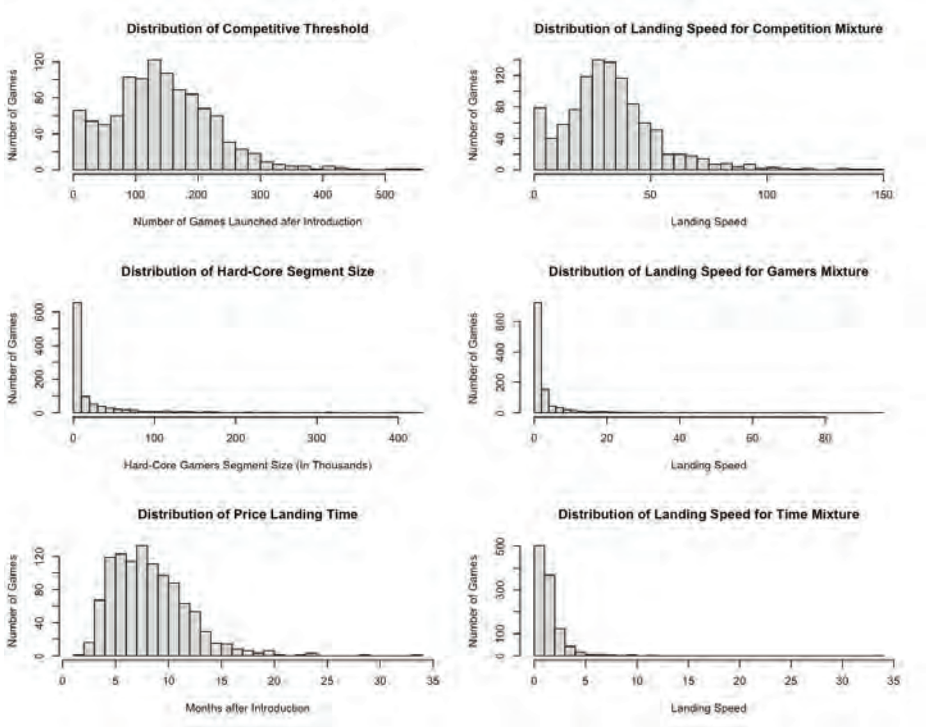

Figure 3.9: Histogram of the Posterior Mean of the Threshold (λ_i^k) and Speed (ν_i^k) Parameters

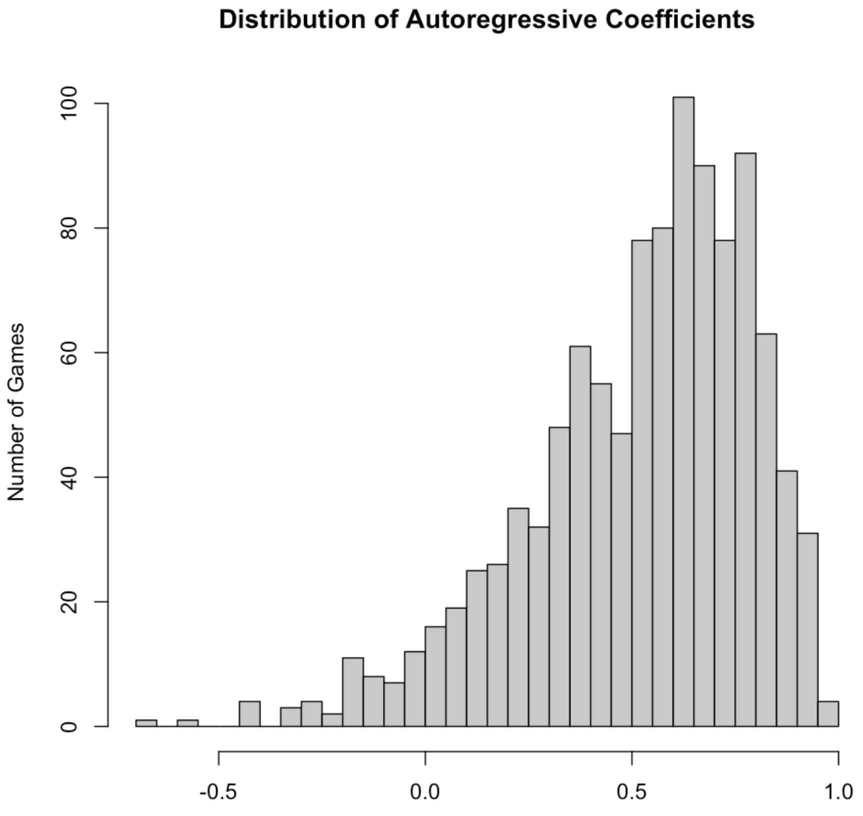

Figure 3.10: Histogram of the Posterior Mean of the α_i Parameters

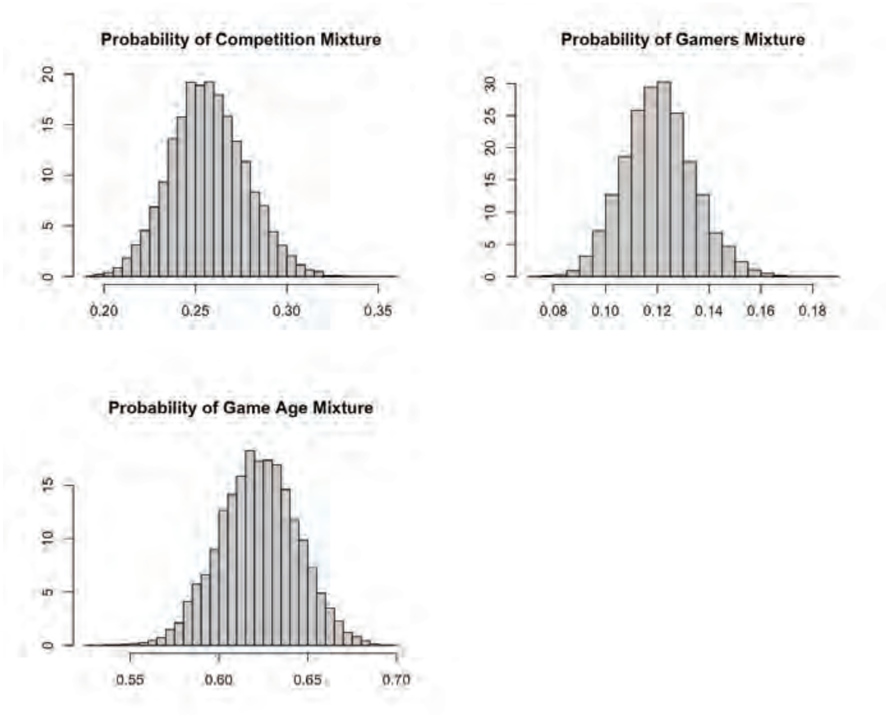

Figure 3.11: Histogram of the Posterior Mean of Price Triggers $P(S_i = k)$

NEW PRODUCTS PRICING STUDIES

Author (Journal, Year)	Approach	Price Change Speed	Price Mimics Diffusion	When to cut prices	Main Price Driver
Bass and Bultez (1982)	Analytical	–	Yes	No	Saturation
Bayus (1994)	Empirical	Gradual	No*	No	Saturation
Bayus (1992)	Analytical	Gradual	Maybe*	No	Learning Curve + Consumer Heterogeneity + Entry
Clements and Ohashi (2005)	Empirical	Gradual	No	No	Saturation + Indirect Network Effects
Chintagunta et al. (2009)	Empirical	Gradual	No	No	Saturation + Marketing Mix
Dockner and Gaunersdorfer (1996)	Analytical	Gradual	No*	No	Saturation + Entry
Dockner and Jorgensen (1988)	Analytical	Gradual	Yes*	No	Learning Curve + Saturation
Dolan and Jeuland (1981)	Analytical	Gradual	Maybe*	No	Learning Curve
Eliashberg and Jeuland (1986)	Analytical	Jumps	No*	No	Entry
Feng and Gallego (1995)	Analytical	Jumps	No	Yes	Saturation
Ferguson and Koenigsberg (2007)	Analytical	Jumps	No	Yes	Deteriorating Inventory
Franza and Gaimon (1998)	Analytical	Gradual	No	Yes	Entry Timing + Saturation + Learning Curve
Gupta and Di Benedetto (2007)	Analytical	Gradual	No	Yes	Entry + Advertising
Gupta et al. (2006)	Analytical	Jumps	No	Yes	Deteriorating Inventory + Consumer Valuations
Horsky (1990)	Empirical	Gradual	Yes*	No	Saturation
Kalish (1985)	Empirical	Gradual	No	No	Advertising
Kalish (1983)	Analytical	Gradual	Yes*	No	Learning Curve + Saturation
Kalish and Lilien (1983)	Analytical	Gradual	Yes*	No	Saturation
Kornish (2001)	Analytical	Gradual	No*	No	Entry (New Product Generations)
Krishnan et al. (1999)	Analytical	Gradual	No	Yes	Saturation
Nair (2007)	Empirical	Jumps	No	Yes	Consumer Heterogeneity + Expectations
Nascimento and Vanhonacker (1993)	Analytical	Gradual	No	No	Consumer Heterogeneity + New Product Generations
Padmanabhan and Bass (1993b)	Analytical	Jumps	No	Yes	Entry + New Product Generations
Parker (1992)	Empirical	–	No	No	Saturation
Rajan et al. (1992)	Analytical	Jumps	Yes	Yes	Saturation
Rao and Bass (1985)	Analytical	Gradual	Yes	–	Learning Curves + Saturation
Raman and Chatterjee (1995)	Analytical	Gradual	Yes	No	Saturation
Robinson and Lakhani (1975)	Analytical	Gradual	No*	No	Saturation
Schmalen (1982)	Analytical	Jumps	No	No	Entry
Simon (1979)	Empirical	Gradual	No	Yes	Saturation
Teng and Thompson (1996)	Analytical	Jumps	No	Yes	Saturation + Quality
Zhao and Zheng (2000)	Analytical	Jumps	No	Yes	Consumer Heterogeneity

Note: *The study supports price skimming

Table 3.1: Literature Review on New Products Pricing

	Mixture (1) D(t)= VG's Age				Mixture (2) D(t) = Cumulative VG's Introductions			
	Landing Time $log(\lambda_i)$		Landing Speed $log(\nu_i)$		Competitive Threshold $log(\lambda_i)$		Landing Speed $log(\nu_i)$	
Intercept	**1.887*****	(0.266)	-0.575	(0.627)	**1.273*****	(0.389)	**2.419****	(0.914)
Game Type								
Action	**-0.234***	(0.122)	-0.172	(0.307)	-0.050	(0.205)	-0.197	(0.378)
Arcade	-0.248	(0.186)	-0.715	(0.500)	-1.909	(1.517)	-2.508	(2.354)
Children	-0.266	(0.217)	-0.250	(0.526)	0.380	(0.286)	**-1.575****	(0.695)
Driving	**-0.350****	(0.133)	-0.400	(0.334)	-0.308	(0.195)	-0.200	(0.393)
Family	**-0.378****	(0.178)	0.459	(0.417)	**0.504***	(0.281)	-0.602	(0.529)
Fighting	**-0.295***	(0.145)	-0.120	(0.347)	-0.218	(0.224)	-0.607	(0.543)
Role playing	-0.056	(0.143)	-0.493	(0.381)	-0.060	(0.207)	-0.060	(0.502)
Shooter	**-0.646*****	(0.134)	-0.006	(0.334)	-0.182	(0.188)	-0.085	(0.357)
Sports	**-0.276****	(0.123)	-0.249	(0.313)	-0.096	(0.179)	-0.056	(0.382)
Strategy	-0.269	(0.176)	-0.383	(0.393)	-0.142	(0.212)	-0.329	(0.470)
Compilations	-0.342	(0.220)	**1.116****	(0.503)	**-2.450*****	(0.473)	**-5.903*****	(1.389)
Publisher								
Electronic Arts	-0.014	(0.079)	**1.226*****	(0.211)	-0.220	(0.133)	-0.022	(0.298)
Acclaim	**-0.196***	(0.105)	**1.106*****	(0.288)	**-0.507*****	(0.162)	0.253	(0.328)
Infogames	-0.234	(0.147)	**1.016*****	(0.327)	**-0.690*****	(0.196)	-0.020	(0.345)
Konami	**-0.387*****	(0.122)	0.446	(0.344)	-0.042	(0.179)	0.144	(0.412)
Activision	**-0.280****	(0.111)	**0.693****	(0.291)	-0.020	(0.201)	0.146	(0.445)
Midway	0.039	(0.124)	**1.665*****	(0.345)	-0.281	(0.221)	0.115	(0.423)
Eidos Interactive	**-0.791*****	(0.123)	**1.434*****	(0.308)	**-0.697*****	(0.187)	-0.248	(0.522)
THQ	**-0.320****	(0.157)	**1.621*****	(0.414)	**-0.693***	(0.336)	0.338	(0.529)
Capcom	-0.109	(0.128)	0.657	(0.410)	-0.108	(0.199)	0.356	(0.416)
Namco	0.218	(0.159)	**2.109*****	(0.457)	**0.416****	(0.206)	-0.648	(0.965)
Agetec	-0.132	(0.183)	**2.147*****	(0.429)	**-4.106*****	(1.190)	**-5.610*****	(1.620)
Interplay	**-0.758*****	(0.130)	**1.470*****	(0.310)	**-1.798*****	(0.423)	-1.436	(1.074)
Hasbro	-0.097	(0.150)	**0.941****	(0.392)	-1.016	(0.970)	0.910	(1.626)
2nd Publishers	**-0.461*****	(0.099)	**1.335*****	(0.254)	**-0.388****	(0.182)	0.154	(0.488)
3rd Publishers	**-0.299*****	(0.096)	**0.942*****	(0.269)	**-0.636*****	(0.142)	0.066	(0.336)
4th Publishers	**-0.399*****	(0.109)	**1.334*****	(0.267)	**-0.386****	(0.176)	0.410	(0.382)
Season								
Feb	-0.183	(0.150)	-0.561	(0.429)	0.014	(0.235)	-0.364	(0.767)
Mar	-0.111	(0.140)	-0.564	(0.389)	-0.125	(0.223)	-0.403	(0.627)
Apr	-0.056	(0.162)	-0.218	(0.452)	-0.421	(0.267)	-0.514	(0.810)
May	-0.142	(0.154)	-0.549	(0.456)	-0.199	(0.247)	0.110	(0.701)
Jun	0.050	(0.154)	**-0.947****	(0.443)	-0.259	(0.253)	-1.087	(0.718)
Jul	-0.224	(0.173)	-0.354	(0.530)	-0.249	(0.265)	-1.023	(0.907)
Aug	0.045	(0.153)	-0.474	(0.397)	-0.154	(0.276)	-0.550	(0.741)
Sep	-0.014	(0.130)	-0.469	(0.387)	0.023	(0.213)	-0.215	(0.673)
Oct	-0.129	(0.137)	-0.402	(0.393)	-0.042	(0.211)	-0.358	(0.677)
Nov	-0.109	(0.128)	-0.393	(0.373)	0.054	(0.194)	-0.309	(0.678)
Dec	-0.229*	(0.140)	-0.420	(0.421)	-0.287	(0.231)	-0.198	(0.789)
Other covariates								
Launch Price	**0.018*****	(0.005)	0.003	(0.012)	**0.048*****	(0.007)	-0.025	(0.019)
Time to Console Launch	-0.005	(0.004)	-0.002	(0.009)	**-0.021*****	(0.006)	**0.027****	(0.011)

Notes: Posterior standard deviation between parentheses. *,**,*** indicate zero is not contained in the 90, 95 and 99% highest posterior density region

Table 3.2: Estimation Results Part I

	Mixture (3) D(t)=Cumulative Sales				All Mixtures			
	Sales Threshold $log(\lambda_i)$		Landing Speed $log(\nu_i)$		Landing Price κ_i		Starting Price ρ_i	
Intercept	2.579	(1.810)	**4.133*****	(1.363)	**17.34*****	(1.510)	**40.79*****	(2.046)
Game Type								
Action	-0.431	(1.179)	0.161	(0.829)	0.751	(0.987)	-1.793	(1.297)
Arcade	-1.171	(1.567)	-1.866	(1.145)	0.543	(1.492)	**-5.244*****	(1.969)
Children	-1.696	(1.887)	0.551	(1.245)	-1.128	(1.456)	**-10.46*****	(2.092)
Driving	0.342	(1.203)	1.040	(0.718)	-0.243	(1.007)	-1.363	(1.299)
Family	0.705	(1.641)	-0.012	(1.243)	-1.111	(1.225)	**-5.322*****	(1.627)
Fighting	-1.416	(1.129)	-0.974	(0.814)	**2.061***	(1.109)	0.378	(1.443)
Role playing	2.021	(1.730)	0.067	(1.356)	**3.162*****	(1.180)	1.033	(1.460)
Shooter	-0.823	(1.230)	0.170	(0.901)	-0.310	(1.056)	2.050	(1.372)
Sports	-1.147	(1.301)	**-1.369***	(0.845)	-0.987	(0.983)	-1.450	(1.301)
Strategy	-1.545	(1.640)	-0.462	(1.271)	2.364*	(1.240)	1.290	(1.592)
Compilations	-0.603	(1.732)	**2.599****	(1.185)	**-7.490*****	(1.534)	**8.880*****	(3.272)
Publisher								
Electronic Arts	**3.395*****	(0.647)	**2.077*****	(0.442)	**1.135***	(0.671)	**1.712****	(0.807)
Acclaim	0.904	(0.782)	0.039	(0.619)	**-1.572***	(0.865)	1.617	(1.124)
Infogames	0.049	(0.664)	0.119	(0.577)	**-1.766****	(0.888)	1.262	(1.289)
Konami	**2.240****	(1.090)	-0.037	(0.656)	**2.535*****	(0.979)	0.586	(1.199)
Activision	**2.886****	(1.078)	**1.683*****	(0.651)	-1.105	(0.962)	-0.439	(1.196)
Midway	1.346	(1.018)	-0.030	(0.875)	-1.617	(1.054)	2.344*	(1.339)
Eidos Interactive	1.674	(1.142)	**1.537*****	(0.747)	**-2.009****	(1.013)	**5.029*****	(1.421)
THQ	**5.719*****	(1.126)	**2.341*****	(0.888)	-0.919	(1.172)	2.005	(1.656)
Capcom	2.154	(1.326)	**2.476*****	(0.869)	1.540	(1.070)	-0.874	(1.324)
Namco	-2.462	(4.770)	-2.518	(2.743)	1.433	(1.554)	2.235	(1.597)
Agetec	0.806	(1.182)	-0.472	(0.714)	**-3.267****	(1.427)	3.487	(2.106)
Interplay	1.134	(1.241)	**2.043*****	(0.814)	-0.778	(1.127)	**9.382*****	(1.741)
Hasbro	0.816	(1.009)	-1.433	(1.460)	**-3.545*****	(1.289)	**-4.731*****	(1.720)
2nd Publishers	0.398	(0.933)	-0.035	(0.909)	**-3.134*****	(0.767)	**1.892***	(1.081)
3rd Publishers	-0.212	(0.597)	0.007	(0.454)	-1.124	(0.702)	**1.947****	(0.943)
4th Publishers	1.356*	(0.779)	0.408	(0.567)	**-3.216*****	(0.815)	0.771	(1.212)
Season								
Feb	-1.274	(2.237)	-0.508	(1.474)	-0.121	(1.254)	-2.178	(1.747)
Mar	0.364	(1.099)	-0.344	(0.890)	**2.038***	(1.123)	**-2.663***	(1.559)
Apr	**-10.56*****	(3.330)	**-8.255*****	(2.689)	**2.306***	(1.375)	0.148	(1.878)
May	**-12.00*****	(2.095)	**-8.820*****	(1.870)	1.499	(1.281)	-1.118	(1.811)
Jun	**-11.45*****	(2.627)	**-8.422*****	(1.864)	**3.648*****	(1.230)	**-3.058***	(1.717)
Jul	**-3.430****	(1.819)	**-3.950*****	(1.306)	1.227	(1.367)	-1.847	(1.926)
Aug	1.129	(1.519)	0.173	(1.202)	0.236	(1.235)	**-3.719****	(1.655)
Sep	0.739	(0.877)	0.840	(0.709)	1.397	(1.088)	-2.537	(1.536)
Oct	0.849	(0.892)	0.652	(0.761)	1.112	(1.124)	-0.832	(1.573)
Nov	0.588	(0.990)	0.597	(0.720)	0.617	(1.073)	-0.965	(1.498)
Dec	0.109	(0.982)	0.358	(0.766)	**2.150***	(1.173)	0.534	(1.682)
Launch Info								
Launch Price	**0.155*****	(0.018)	**0.068*****	(0.015)	–	–	–	–
Time to Console Launch	**-0.122*****	(0.025)	**-0.059*****	(0.018)	**0.127*****	(0.025)	**0.401*****	(0.033)

Notes: Posterior standard deviation between parentheses. *,**,*** indicate zero is not contained in the 90, 95 and 99% highest posterior density region

Table 3.3: Estimation Results Part II

	Latent Utility of Sales Mixture		Latent Utility of Time Mixture	
Intercept	3.727***	(0.579)	0.549	(0.445)
Game Type				
Action	0.699*	(0.360)	0.260	(0.269)
Arcade	-0.032	(0.543)	0.404	(0.451)
Children	0.481	(0.479)	0.005	(0.456)
Driving	0.708*	(0.402)	0.288	(0.278)
Family	-0.100	(0.451)	-0.577*	(0.333)
Fighting	1.153***	(0.427)	0.357	(0.372)
Role playing	-0.304	(0.552)	-0.146	(0.355)
Shooter	0.809	(0.567)	0.716**	(0.297)
Sports	-0.010	(0.372)	0.057	(0.252)
Strategy	0.342	(0.613)	-0.055	(0.362)
Compilations	0.930*	(0.530)	-0.258	(0.438)
Publisher				
Electronic Arts	-0.689	(0.514)	-0.137	(0.239)
Acclaim	-0.723	(0.583)	-0.365	(0.284)
Infogames	0.723	(0.456)	-0.332	(0.360)
Konami	1.110**	(0.469)	0.068	(0.343)
Activision	1.060**	(0.450)	0.356	(0.325)
Midway	0.884*	(0.454)	-0.119	(0.384)
Eidos Interactive	0.002	(0.668)	0.265	(0.365)
THQ	-0.287	(0.531)	-0.017	(0.459)
Capcom	-0.355	(0.451)	-0.821**	(0.357)
Namco	-0.043	(0.649)	-0.503	(0.426)
Agetec	0.592	(0.558)	0.592	(0.512)
Interplay	0.648	(0.626)	0.800*	(0.430)
Hasbro	-0.364	(0.512)	0.303	(0.380)
2nd Publishers	0.203	(0.427)	0.387	(0.305)
3rd Publishers	0.324	(0.402)	-0.025	(0.272)
4th Publishers	0.510	(0.374)	0.164	(0.301)
Season				
Feb	0.159	(0.467)	-0.026	(0.361)
Mar	0.002	(0.454)	0.441	(0.297)
Apr	0.448	(0.575)	0.296	(0.431)
May	-0.309	(0.499)	-0.361	(0.344)
Jun	0.067	(0.533)	0.709**	(0.381)
Jul	0.149	(0.547)	0.007	(0.417)
Aug	-0.197	(0.479)	0.355	(0.367)
Sep	0.008	(0.373)	-0.214	(0.259)
Oct	0.156	(0.410)	0.285	(0.293)
Nov	0.070	(0.357)	0.069	(0.243)
Dec	0.666	(0.418)	0.329	(0.307)
Launch Info				
Launch Price	-0.162***	(0.013)	0.004	(0.007)
Time to Launch	0.010	(0.015)	-0.012	(0.008)

Notes: Posterior standard deviation in parentheses. *,**,*** indicate zero is not contained in the 90, 95, and 99% highest posterior density region.

Table 3.4: Results of Hierarchical Structure for Mixture Probabilities

Game Title	Forecasted Months	St. Dev. Price	Forecast RMSE	Forecast RMSE AR(1)	Log of Predicted Density	Log Likelihood of predicted AR (1)
NHL 2001	10	0.18	0.17	0.15	-0.89*	-3.47
JJ'S VR FOOTBALL 98	8	2.39	1.76*	5.19	-5.06*	-6.79
HIGH HEAT BSBALL 2002	18	7.14	2.18*	11.92	-10.25*	-142.23
MADDEN NFL 98	12	4.56	2.55*	5.03	-11.05*	-27.81
MR DOMINO	18	8.33	3.10*	12.34	-17.22*	-188.45
THE CROW CITY ANGELS	18	14.53	3.24*	26.27	-12.15*	-369.19
PITBALL	18	13.49	3.72*	27.92	-12.50*	-264.59
FROGGER 2	18	10.48	3.86*	22.53	-12.30*	-2421.81
BIG OL' BASS 2	18	14.45	3.92*	22.63	-15.22*	-485.79
MK & ASHLEY WINNER'S	18	11.88	4.11*	17.46	-11.90*	-493.25
CIVILIZATION 2	18	9.73	4.25*	12.52	-8.94*	-1009.23
PONG	18	11.43	4.37*	20.81	-14.54*	-646.32
ROGUE TRIP	14	1.80	4.38	1.90	-16.24	-0.58
RESIDENT EVIL 3:NEMES	18	10.16	4.70*	10.34	-15.91*	-71.64
ETERNAL EYES	18	8.31	4.92*	9.45	-24.18*	-82.53
TEKKEN 2	18	7.39	5.13*	8.54	-15.13*	-85.26
TEST DRIVE 4	18	11.50	5.39*	28.29	-12.53*	-511.42
F1 WRLD GRAND PRIX 00	18	7.11	5.63*	7.52	-29.76*	-50.37
FADE TO BLACK	18	9.09	5.88*	8.58	-21.99*	-38.53
SHEEP RAIDER	18	9.35	6.03*	11.20	-81.57*	-112.25
G POLICE2:WPN JUSTICE	9	10.79	6.04*	24.87	-8.60*	-386.84
RISK	10	9.50	6.55*	12.39	-13.02*	-267.31
SYNDICATE WARS	18	8.93	6.66*	12.83	-16.71*	-55.53
JUGGERNAUT	18	9.33	6.71*	16.33	-51.62*	-60.97
KISS PINBALL	10	8.47	6.73*	13.79	-11.21*	-128.41
BACKYARD SOCCER	18	16.59	6.74*	23.35	-24.94*	-652.10
OLYMPIC SUMMER GAMES	18	8.57	7.02*	12.52	-16.79*	-38.77
NECTARIS:MILITARY MAD	18	13.34	7.06*	19.63	-16.75*	-292.34
T.CLANCYS ROGUE SPEAR	18	5.38	7.88	4.54	-21.11	-16.62
TOCA 2 CAR CHALLENGE	18	13.75	7.97*	13.93	-23.36*	-177.86
NFL XTREME 2	18	14.35	8.27*	24.53	-20.69*	-467.21
ARENA FOOTBALL	17	3.40	8.35	4.08	-14.76*	-23.53
FINAL FANTASY IX	13	6.23	8.43*	12.81	-10.78*	-32.38
SHEEP	18	3.47	8.83	3.54	-22.08	-4.11
SIMPSON'S WRESTLING	12	8.66	8.87*	12.12	-16.69*	-31.70
POCKET FIGHTER	18	10.51	9.02*	17.25	-17.09*	-169.35
POWERBOAT RACING	18	10.50	9.19*	24.08	-14.13*	-466.81
GRAND SLAM 97	18	11.09	9.53*	11.69	-24.66*	-121.27
RAMPAGE WORLD TOUR	6	2.88	9.67	4.58	-1245.7	-6.05
EAGLE ONE: HARRIER	13	11.43	10.5*	13.02	-50.33*	-847.83
STRIKER PRO 2000	9	10.66	10.6*	20.87	-10.52*	-139.91
NEWMAN/HAAS RACING	16	3.88	11.31	4.63	-38.44	-4.55
DISCWRLD 2:MRTLY BYTE	18	6.31	11.42	5.95	-87.15*	-41.79
CROSSROAD CRISIS	18	9.77	12.4*	15.33	-93.93*	-458.48
SLAM N JAM 96	18	10.23	13.3*	19.53	-18.82*	-280.41
NBA LIVE 2002	18	9.23	14.4*	21.76	-80.22*	-466.39
ARMD COR 2 PRJ PNTSMA	15	8.71	15.0*	16.21	-11.77*	-263.19
CRASH TEAM RACING	18	15.92	15.8*	17.91	-342.07	-117.05
DISNEY'S DINOSAUR	18	5.00	16.35	5.68	-27.82	-15.57
NFL BLITZ 2000	18	5.63	17.57	6.41	-30.46*	-130.64

Notes: * Means the RMSE or the predictive likelihood is smaller in our model than in the AR(1)

Table 3.5: Forecasting Performance

Game Title	Forecast Horizon	St.Dev. Price[a]	Log of Predictive Density (LPD) Original Model	Log of Predicted Density (LDP) Alt. Model
NHL 2001	10	0.18	-0.89 *	-0.90
JJ'S VR FOOTBALL 98	8	2.39	-5.06 *	-4.89
HIGH HEAT BSBALL 2002	18	7.14	-10.25 *	-9.92
MADDEN NFL 98	12	4.56	-11.05 *	-11.12
MR DOMINO	18	8.33	-17.22	-15.86
THE CROW CITY ANGELS	18	14.53	-12.15 *	-11.29
PITBALL	18	13.49	-12.50 *	-12.06
FROGGER 2	18	10.48	-12.30 *	-12.51
BIG OL' BASS 2	18	14.45	-15.22 **	-16.29
MK & ASHLEY WINNER'S	18	11.88	-11.90 *	-12.11
CIVILIZATION 2	18	9.73	-8.94 **	-11.36
PONG	18	11.43	-14.54	-13.10
ROGUE TRIP	14	1.80	-16.24 **	-30.14
RESIDENT EVIL 3:NEMES	18	10.16	-15.91 **	-17.48
ETERNAL EYES	18	8.31	-24.18 **	-25.35
TEKKEN 2	18	7.39	-15.13 **	-16.17
TEST DRIVE 4	18	11.50	-12.53 *	-12.40
F1 WRLD GRAND PRIX 00	18	7.11	-29.76	-26.22
FADE TO BLACK	18	9.09	-21.99 **	-23.64
SHEEP RAIDER	18	9.35	-81.57 **	-121.60
G POLICE2:WPN JUSTICE	9	10.79	-8.60 *	-8.56
RISK	10	9.50	-13.02 **	-33.94
SYNDICATE WARS	18	8.93	-16.71	-13.63
JUGGERNAUT	18	9.33	-51.62 **	-98.83
KISS PINBALL	10	8.47	-11.21 *	-11.05
BACKYARD SOCCER	18	16.59	-24.94	-21.56
OLYMPIC SUMMER GAMES	18	8.57	-16.79	-14.71
NECTARIS:MILITARY MAD	18	13.34	-16.75 *	-16.17
T.CLANCYS ROGUE SPEAR	18	5.38	-21.11 **	-22.90
TOCA 2 CAR CHALLENGE	18	13.75	-23.36 **	-26.08
NFL XTREME 2	18	14.35	-20.69	-17.40
ARENA FOOTBALL	17	3.40	-14.76 *	-14.56
FINAL FANTASY IX	13	6.23	-10.78	-9.54
SHEEP	18	3.47	-22.08 **	-23.30
SIMPSON'S WRESTLING	12	8.66	-16.69 *	-16.84
POCKET FIGHTER	18	10.51	-17.09 *	-16.58
POWERBOAT RACING	18	10.50	-14.13 *	-14.76
GRAND SLAM 97	18	11.09	-24.66 *	-23.81
RAMPAGE WORLD TOUR	6	2.88	-1245.7 **	-2072.64
EAGLE ONE: HARRIER	13	11.43	-50.32 **	-77.20
STRIKER PRO 2000	9	10.66	-10.52 *	-11.26
NEWMAN/HAAS RACING	16	3.88	-38.44 **	-54.93
DISCWRLD 2:MRTLY BYTE	18	6.31	-87.15	-38.19
CROSSROAD CRISIS	18	9.77	-93.93	-34.08
SLAM N JAM 96	18	10.23	-18.82	-15.18
NBA LIVE 2002	18	9.23	-80.22 **	-82.06
ARMD COR 2 PRJ PNTSMA	15	8.71	-11.77 **	-14.41
CRASH TEAM RACING	18	15.92	-342.07 **	-397.18
DISNEY'S DINOSAUR	18	5.00	-27.82	-18.46
NFL BLITZ 2000	18	5.63	-30.46 **	-34.48

Notes:** means that the Original Model LPD is greater than the Alternative LPD by more than 1 unit, * means that the difference between the original and alternative are less than 1 unit. Alt. stand for Alternative and St.Dev for Standard Deviation.

Table 3.6: Comparison with Alternative Model

3.A Estimation Methodology

To draw inference on the parameters we will rely on a Bayesian analysis and more specifically the Gibbs sampler. Whenever possible we use Gibbs sampling with block updating and when there are no closed form sampling distributions we rely on the Metropolis algorithm. We run a Markov Chain for 200 thousand iterations of which the first 100 thousand are discarded for burn-in and we keep each tenth remaining draws. This Markov Chain has the posterior distribution of the parameters and the latent trigger variable indicators S_i $i = 1, \ldots, N$ as the stationary distribution. We programmed all our routines in Ox (see Doornik (2007)) and our graphs in R (see R Development Core Team (2005)).

In all what follows we collect the first level model parameters in the blocks: $\tau_i = (\rho_i, \kappa_i, \alpha_i, \sigma_i, \lambda_k, \nu_k)$, $\rho = (\rho_1, ..., \rho_N)$, $\kappa = (\kappa_1, ..., \kappa_N)$, $\alpha = (\alpha_1, ..., \alpha_N)$, $\sigma^2 = (\sigma_1^2, ..., \sigma_N^2)$, $\lambda_k = (\ln(\lambda_{ki}), ...,\ln(\lambda_{kN}))$ and finally $\nu_k = (\ln(\nu_{ik}), ..., \ln(\nu_{Nk}))$.

We further collect all hyper-parameters in the following blocks: $\theta = (\gamma^P, \gamma^L, \Pi, \mathbf{\Omega})$, where $\mathbf{\Omega} = (\mathbf{\Omega_1}, ..., \mathbf{\Omega_K})$, $\Pi = (\pi_1, ..., \pi_K)$. We have that $\gamma^P = (\gamma^\kappa, \gamma^\rho)$ where $\gamma^\kappa = (\gamma_1^\kappa, ..., \gamma_M^\kappa)$ and $\gamma^\rho = (\gamma_1^\rho, ..., \gamma_M^\rho)$. Finally, $\gamma^L = (\gamma_1^L, \ldots, \gamma_K^L)$ where $\gamma_k^L = (\gamma_k^\lambda, \gamma_k^\nu)$ and $\gamma_k^\lambda = (\gamma_{k1}^\lambda, ..., \gamma_{kM}^\lambda)$ and $\gamma_k^\nu = (\gamma_{k1}^\nu, ..., \gamma_{kM}^\nu)$. M refers to the number of variables in Z, K refers to the number of mixtures (same as number of triggers), and N refers to the total number of products. Next $\mathbf{Z}$=$(Z_1, ..., Z_M)$ and $\phi(x; \mu, \sigma^2)$ denotes the normal pdf distribution with mean μ and variance σ^2 evaluated at x. Finally, $p()$ denotes a general density function and $IW(\widehat{\Omega}, N)$ denotes the inverted Whishart distribution with scale matrix $\widehat{\Omega}$ and N degrees of freedom.

Note that in this context we treat the product specific *parameters* τ_i as latent variables. We consider the log of λ_{ki} and ν_{ki} $k = 1, ...K$, $i = 1, \ldots, N$ as focal parameters strictly for convenience and to impose that λ_{ki} and ν_{ki} are positive. This has no impact on the results. In this Markov Chain we will sample the latent variables alongside the parameters.

The complete data likelihood for product i is

$$p(\mathbf{P}_i, S_i, \tau_i | \theta) = \pi_{S_i} \times p(\mathbf{P}_i | S_i, \tau_i, \theta) \times p(\tau_i | \theta), \tag{3.12}$$

where $\mathbf{P}_i = (P_i(0), ..., P_i(T))$ and $p(\mathbf{P}_i|S_i, \tau_i, \theta)$ is equal to

$$p(P_i(0)|S_i, \tau_i, \theta) \times \prod_{t=1}^{t=T} p(P_i(t)|P_i(t-1), S_i, \tau_i, \theta). \tag{3.13}$$

Furthermore, we have that the first observation likelihood is

$$p(P_i(0)|S_i, \tau_i, \theta) = \phi\left(P_i(0); P_i^*(0), \frac{1}{1-\alpha^2}\sigma_i^2\right), \tag{3.14}$$

and all other observations have as likelihood

$$p(P_i(t)|P_i(t-1), S_i, \tau_i, \theta) = \phi\left(P_i(t); P_i^*(t) + \alpha_i[P_i(t-1) - P_i^*(t-1)], \sigma_i^2\right). \tag{3.15}$$

Next, we have

$$p(\tau_i|\theta) = p(\rho_i, \kappa_i|\theta) \prod_{k=1}^{K} p(\lambda_{ki}, \nu_{ki}|\theta), \tag{3.16}$$

where

$$p\left((\rho_i, \kappa_i)|\theta\right) = \phi\left((\rho_i, \kappa_i)'; \gamma^{P\prime} Z_i, \mathbf{\Sigma}\right), \tag{3.17}$$

and

$$p\left((\lambda_{ki}, \nu_{ki})|\theta\right) = \phi\left((\lambda_{ki}, \nu_{ki})'; \gamma_k^{L\prime} Z_i, \mathbf{\Omega}_k\right). \tag{3.18}$$

We impose flat priors on all almost all parameters, for α_i we set a uniform prior on the interval (-1,1) to impose stationarity. This completes the main model specification and next we discuss how we sample from the posterior distribution for all parameters.

Sampling distributions

If π_k is fixed across products, the density of S_i conditional on $\mathbf{P_i}$, τ_i, and θ equals a Multinomial distribution with probabilities proportional to

$$\pi_{S_i} \times p(\mathbf{P}_i|S_i, \tau_i, \theta). \tag{3.19}$$

The full conditional distribution for α_i is a truncated normal on the interval [-1,1], where the mean and variance are given by applying the Ordinary Least Squares formulas to a regression of $P_i(t)$-$P_i^*(t)$ on its lag with known variance of the disturbance term σ_i^2. A draw for σ_i^2 can be obtained using the Metropolis-Hastings sampler and taking as candidate

$$\sigma^2_{i_{cand}} = \frac{\sum_{t=1}^{T} (\hat{\varepsilon}_i(t))^2}{w} \quad \text{where} \quad w \sim \chi^2_{(T-1)}, \tag{3.20}$$

where $\hat{\varepsilon}_i(t)$ is the residual of equation (3.4) given all other parameters. We evaluate this candidate and the current draw of σ_i^2 in the conditional distribution of the first observation given in equation (3.14). Hence we take the candidate as the next drawn value of σ_i^2 with probability

$$\min\left(1, \frac{\phi\left(P_i(0); P_i^*(0), \frac{1}{1-\alpha^2}\sigma^2_{i_{cand}}\right)}{\phi\left(P_i(0); P_i^*(0), \frac{1}{1-\alpha^2}\sigma^2_{i_{current}}\right)}\right). \tag{3.21}$$

To derive the full conditional distribution of κ_i and ρ_i we first rewrite equations (3.4) and (3.5) as

$$\sqrt{1-\alpha_i^2}P_i(0) = [\sqrt{1-\alpha_i^2}h_{S_i}(0)] \times \kappa_i + [\sqrt{1-\alpha_i^2}h_{S_i}(0)] \times \rho_i + \varepsilon_i(0), \tag{3.22}$$

and

$$P_i(t) - \alpha_i P_i(t-1) = [1 - h_{S_i}(t) - \alpha_i(1 - h_{S_i}(t))] \times \kappa_i + [h_{S_i}(t) - \alpha_i h_{S_i}(t)] \times \rho_i + \varepsilon_i(t). \tag{3.23}$$

These equations should be combined with the specification of the hierarchical layer in (3.7) as follows:

$$\begin{pmatrix} Y_i \\ \gamma^{\rho'} Z_i \\ \gamma^{\kappa'} Z_i \end{pmatrix} = \begin{pmatrix} X_i^A & X_i^B \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_i \\ \kappa_i \end{pmatrix} + \begin{pmatrix} \varepsilon_i \\ \omega^\rho \\ \omega^\kappa \end{pmatrix}, \tag{3.24}$$

where we define X_i^A and X_i^B as

$$X_i^A = \begin{pmatrix} \sqrt{1-\alpha_i^2}(1-h_{S_i}(0)) \\ 1-h_{S_i}(1)-\alpha_i(1-h_{S_i}(1)) \\ \vdots \\ 1-h_{S_i}(T_i)-\alpha_i(1-h_{S_i}(T_i)) \end{pmatrix} \quad \text{and} \quad X_i^B = \begin{pmatrix} \sqrt{1-\alpha_i^2}h_{S_i}(0) \\ h_{S_i}(1)-\alpha_i h_{S_i}(1) \\ \vdots \\ h_{S_i}(T_i)-\alpha_i h_{S_i}(T_i) \end{pmatrix}, \tag{3.25}$$

and Y_i as

$$Y_i = \begin{pmatrix} \sqrt{1-\alpha_i^2}P_i(0) \\ P_i(1)-P_i(0) \\ \vdots \\ P_i(T)-P_i(T-1) \end{pmatrix}. \tag{3.26}$$

Finally, we can draw κ_i and ρ_i from

$$N\left((W_i'\Gamma_i^{-1}W_i)^{-1}W_i'\Gamma_i^{-1}Y_i, (W_i'\Gamma^{-1}W_i)^{-1}\right), \tag{3.27}$$

where

$$W_i = \begin{pmatrix} X_i^A & X_i^B \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad E\left(\begin{pmatrix} \varepsilon_i & \omega^\rho & \omega^\kappa \end{pmatrix}\begin{pmatrix} \varepsilon_i \\ \omega^\rho \\ \omega^\kappa \end{pmatrix}\right) = \begin{pmatrix} \sigma_i^2 I & 0 \\ 0 & \mathbf{\Sigma} \end{pmatrix} = \Gamma_i. \tag{3.28}$$

Due to the non-linearity in the price patterns, the conditional distributions of λ_k and ν_k are not of a known form. We will sample each parameter one at a time using a random walk Metropolis Hastings sampler. Given the current draw of one of these parameters we draw a candidate by adding a draw from a normal with mean zero and a fixed variance. This candidate draw for λ_k and ν_k is accepted with probability

$$\min\left(1, \frac{p(\lambda_{ki}^{cand}|\nu_{ki})}{p(\lambda_{ki}^{current}|\nu_{ki})}\right) \quad \text{and} \quad \min\left(1, \frac{p(\nu_{ki}^{cand}|\lambda_{ki})}{p(\nu_{ki}^{current}|\lambda_{ki})}\right), \tag{3.29}$$

respectively. The posterior of the $i'th$ element of λ_k is

$$p(\lambda_{ki}|\nu_{ki}) = p(P_i(0)|S_i, \tau_i, \theta) \prod_{t=1}^{t=T} p(P_i(t)|P_i(t-1)S_i, \tau_i, \theta)\phi\left(\lambda_{ki}; \lambda_{ki}|\nu_{ki}, \Omega_k^{\lambda_{ki}|\nu_{ki}}\right), \tag{3.30}$$

and the posterior of the $i'th$ element of ν_k is

$$p(\nu_{ki}|\lambda_{ki}) = p(P_i(0)|S_i, \tau_i, \theta) \prod_{t=1}^{t=T} p(P_i(t)|P_i(t-1)S_i, \tau_i, \theta)\phi\left(\nu_{ki}; \nu_{ki}|\lambda_{ki}, \Omega_k^{\nu_{ki}|\lambda_{ki}}\right). \tag{3.31}$$

Here $x|y$ refers to the conditional mean of x given y and $\sigma^{x|y}$ refers to the conditional variance of x given y. These are conditional posterior distributions because we allow λ_k and ν_k to be correlated to each other. In other words, the timing of the price cut and the speed of the price cut might be correlated and these correlation is different across mixtures. The variance of the proposal density is chosen such that we obtain an acceptance rate close to approximately 25%, that is the optimal rate for high-dimensional models (see Robert and Casella (2004, page 316), Carlin and Louis (2000, page 154) or Gamerman and Lopes (2006, page 196)).

The conditional distribution of $\pi_1, \ldots, \pi_K$ is a Dirichlet distribution with parameters $1 + \sum_i 1[S_i = 1], \ldots, 1 + \sum_i 1[S_i = K]$; that is, we draw each π_k proportional to the number of products assigned to mixture k, that is $\sum_1 1[S_i = k]$, and naturally restrict $\sum_k \pi_k{=}1$.

Given the latent variables in τ_i sampling the hyper-parameters of the hierarchical part for the marginal costs, launch price, and price landing characteristics is relatively straightforward. We draw γ^P from a normal

$$\gamma^P \sim N\left(\begin{array}{c} (\mathbf{Z}'\mathbf{Z})^{-1}Z'\kappa \\ (Z'\mathbf{Z})^{-1}\mathbf{Z}'\rho \end{array}, \boldsymbol{\Sigma} \otimes (\mathbf{Z}'\mathbf{Z})^{-1}\right), \tag{3.32}$$

and $\gamma_k^L|\Omega_k$ from

$$N\left(\begin{array}{c} \frac{1}{1+g}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\lambda_{ki} \\ \frac{1}{1+g}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\nu_{ki} \end{array}, \frac{1}{1+g}\boldsymbol{\Omega}_k \otimes (\mathbf{Z}'\mathbf{Z})^{-1}\right). \tag{3.33}$$

The factor g comes from the g-prior which states that the variance of (λ_{ki}, ν_{ki}) is proportional to the variance of the data. See Fernandez et al. (2001) for a detailed discussion.

Finally, we draw $\mathbf{\Sigma} \sim IW\left(\widehat{\mathbf{\Sigma}}, N\right)$ where

$$\widehat{\mathbf{\Sigma}} = \begin{pmatrix} \widehat{\omega}^{\kappa} \\ \widehat{\omega}^{\rho} \end{pmatrix} (\widehat{\omega}^{\kappa}, \widehat{\omega}^{\rho}) \tag{3.34}$$

and $\widehat{\omega}_i^{\kappa} = \kappa_i - Z_i'\gamma^{\kappa}$, $\widehat{\omega}_i^{\rho} = \rho_i - Z_i'\gamma^{\rho}$ and $\widehat{\omega}^{\kappa} = (\widehat{\omega}_1^{\kappa} \ldots \widehat{\omega}_N^{\kappa})$ and $\widehat{\omega}^{\rho} = (\widehat{\omega}_1^{\rho} \ldots \widehat{\omega}_N^{\rho})$. Next, we draw $\mathbf{\Omega_k} \sim IW\left(\widehat{\mathbf{\Omega}}_{\mathbf{k}} + \mathbf{G} + I_2, 7 + N\right)$ where

$$\widehat{\mathbf{\Omega}}_{\mathbf{k}} = \begin{pmatrix} \widehat{\eta}_k^{\lambda} \\ \widehat{\eta}_k^{\nu} \end{pmatrix} (\widehat{\eta}_k^{\lambda}, \widehat{\eta}_k^{\nu}), \tag{3.35}$$

and $\widehat{\eta}_k^{\lambda} = \log(\lambda_k) - Z'\gamma_k^{\lambda}$ and $\widehat{\eta}_k^{\nu} = log(\nu_k) - Z'\gamma_k^{\nu}$. finally $\mathbf{G}_k$ is defined as

$$\widehat{\mathbf{G}}_k = \begin{pmatrix} \widehat{\gamma}_k^{\lambda} \\ \widehat{\gamma}_k^{\nu} \end{pmatrix} g(\mathbf{Z}'\mathbf{Z})^{-1} (\widehat{\gamma}_k^{\lambda}, \widehat{\gamma}_k^{\nu}) \tag{3.36}$$

and I_2 is an identity matrix size 2×2.

Hierarchical Structure in the Mixture Probabilities

The previous steps give the methodology to analyze our model without a hierarchical specification on the mixture probabilities π_k. As discussed in this chapter, the model can be easily expanded to include a hierarchical specification on the mixture probabilities. As before, we will assume that π_{ki} differs across products but here we test if a multinomial probit specification that depends on $\mathbf{Z}$ is useful to explain their heterogeneity. For that we need to define first K latent variables for each product i

$$y_{ki}^{*} \sim N(Z_i'\delta_k, 1) \tag{3.37}$$

where $\delta_1 = 0$ for identification. Product i belongs to mixture m if y_{mi}^{*} is the largest of all y_{ki}^{*} $k = 1, \ldots, K$. Given (3.37) , we can write the conditional distribution of y_{mi}^{*} given the other latent utilities (-m), denoted as $y^*_{-m,i}$, as follows:

$$\begin{aligned} p(y^*_{mi}|y^*_{-m,i},\theta,\tau_i,S_i) = \quad & p(y^*_{mi} > \max(y^*_{-m,i})) \times p(\mathbf{P}_i, S_i = m, \tau_i|\theta) \\ & + p(y^*_{mi} < \max(y^*_{-m,i})) \times p(\mathbf{P}_i, S_i = m^*, \tau_i|\theta) \end{aligned} \tag{3.38}$$

where $m^* = \underset{m \neq k}{argmax}(y^*_{ki})$. Based on (3.38) we can apply the inverse cdf technique to draw y^*_{mi} from its full conditional distribution. Note that in this specification the indicator variable S_{ki} is determined based on y^*_{mi} and the δ_m parameters can be obtained from a normal with mean $(Z_i'Z_i)^{-1}Z_i'\delta_k$ and variance $(Z_i'Z_i)^{-1}$ for $m = 2, \ldots, K$.

Posterior Predictive Density

We used two measures to compare predictive performance in Table 3.5: the root mean squared error and the log of the posterior predictive density for observations after $t = 7$. The predictive density $\log(p(P_i(7)_{,\ldots,}P_i(T)|P_i(1)_{,\ldots,}P_i(6)))$ is defined as:

$$\log \int\int\int p(P_i(7)_{,\ldots,}P_i(T)|P_i(1)_{,\ldots,}P_i(6), S_i, \tau_i, \theta) \times p(S_i, \tau_i, \theta|P_i(1)_{,\ldots,}P_i(6))dS_i d\tau_i d\theta \tag{3.39}$$

That is, we compute the log of the density for the forecast sample given the six observations included in the model and the posterior of all model parameters given these latter observations. The posterior predictive density can easily be obtained from the MCMC output by taking the log of the average out-of-sample likelihood over all draws.

Chapter 4

Random Coefficient Logit Models for Large Datasets

We present an approach for analyzing market shares and products´ price elasticities based on large datasets containing aggregate sales data for many products, several markets and for relatively long time periods. We consider the recently proposed Bayesian approach of Jiang et al [Jiang, Renna, Machanda, Puneet and Peter Rossi, 2009. Journal of Econometrics 149 (2) 136-148] and we extend their method in four directions. First, we reduce the dimensionality of the covariance matrix of the random effects by using a factor structure. The dimension reduction can be substantial depending on the number of common factors and the number of products. Second, we parametrize the covariance matrix in terms of correlations and standard deviations, like Barnard et al. [Barnard, John, McCulloch, Robert and Xiao-Li Meng, 2000. Statistica Sinica 10 1281-1311] and we present a Metropolis sampling scheme based on this specification. Third, we allow for long term trends in preferences using time-varying common factors. Inference on these factors is obtained using a simulation smoother for state space time series. Finally, we consider an attractive combination of priors applied to each market and globally to all markets to speed up computation time. The main advantage of this prior specification is that it let us estimate the random coefficients based on all data available. We study both simulated data and a real dataset containing several markets each consisting of 30 to 60 products and our method proves to be promising with immediate practical applicability.

4.1 Introduction

A growing number of scholars is developing estimation methods for random coefficient logit models based on aggregate sales data. Currently, the estimation methods are based on the generalized method of moments [GMM], as in Nevo (2001) and Berry et al. (1995) (hereafter BLP), or based on likelihood or Bayesian approaches, as in Jiang et al. (2009) (hereafter Bayesian BLP or BBLP), Yang et al. (2003), and Park and Gupta (2009). The choice of the estimation method depends on the modeling assumptions regarding aggregate demand shocks, consumer heterogeneity, stability of preferences, price endogeneity and on the size and type of data available.

Recent Bayesian and maximum likelihood-based approaches have been successfully applied to data containing relatively long time series of weekly data (ranging from one to six years) concerning a small number of products (usually less than 6 products) sold in a single market (Jiang et al., 2009; Musalem et al., 2006; Yang et al., 2003). The GMM approach has been applied to similar sized data, like in Goeree (2008). Recently, Berry and Pakes (2007) use GMM and apply an extension of the BLP model to data consisting of both small (between 2 and 10) and large (100) number of products. The extension of Berry and Pakes (2007) is mainly focused on relaxing the assumption of non-zero demand shocks specifically when the market is saturated with many products. Their specification of null demand shocks may decrease the precision of the BLP contraction mapping and they present new complementary routines that overcome this issue.

One of the most challenging aspects for all methods is the estimation of the underlying distribution of the random effects that describe individual level consumer heterogeneity. As only aggregate data is available, the heterogeneity needs to be identified based on switching patterns. The simulation results of Jiang et al. (2009) suggest that their Bayesian method performs well and makes a more efficient use of the data relative to a GMM estimator. Nonetheless, today still little is known about the scalability (that is the performance and adaptability) of current methodologies to settings with many products and markets.

In this chapter we investigate the scalability of the Bayesian approach proposed by Jiang et al. (2009) and we extend their method in four directions. First, we propose a factor structure for the covariance matrix of the random effects. We will assume that the covariance matrix between J products can be modeled by a group of K factors, where the factor loadings are based on observable characteristics. Such a structure helps to keep the dimension of the heterogeneity structure under control. That is, we make the same distributional assumptions as in Jiang et al. (2009) regarding the heterogeneity and aggregate demand shocks but we strongly reduce the dimension of the covariance matrix. This reduction will be especially important in applications with a large number of products.

Second, we specify the covariance matrix following Barnard et al. (2000) as a function of correlations and standard deviations and we propose a Metropolis sampling scheme based on this parametrization. This parametrization has two main advantages. A technical advantage is that splitting the covariance in variances and correlations allows for a more efficient sampling scheme. A practical advantage is that the correlation structure of the heterogeneity itself may be most informative for managers.

The third extension in our approach is that we allow for time variation in preferences. Preference fluctuations are likely to occur over long periods of time and over seasons. In the currently considered Random Coefficient Logit Models such developments are often ignored. One exception we are aware of is Chintagunta et al. (2005), who show that allowing for time variation in preferences is beneficial to reduce both the uncertainty regarding brand preferences and the uncertainty regarding the sensitivity of products' shares to marketing efforts.

Finally, we consider an attractive combination of priors applied to each market and globally to all markets. This prior specification let us analyze all data simultaneously and it facilitates the estimation of the underlying distribution of the random coefficients based in all the data.

The Bayesian approach we use in this chapter allows for an efficient implementation of the four extensions mentioned above. One main advantage of the Bayesian approach over simulated maximum likelihood and GMM is that inference over any function of the parameters is straightforward because we obtain the posterior distribution of all parameters as the MCMC output. This for example allows for a straightforward assessment of the uncertainty in (cross) price elasticities. A second main advantage of the Bayesian approach is that we can incorporate efficient sampling of time-varying parameters alongside the other model parameters. Chintagunta et al.

(2005) use MLE and specify brand-specific time-fixed effects to account for time variation in preferences. Their specification of brand and time-specific fixed effects is an attractive formulation but the number of fixed effects to estimate may increase rapidly as the number of brands and time periods increases. As Chintagunta et al. (2005), we allow for time-variation in preferences but we use the simple and efficient simulation smoother of Durbin and Koopman (2002) to sample the time-varying parameters. The smoother is flexible because it let us reduce the model to the setting where brand preferences are fixed in time and this reduction may depend on the model's parameter estimates or it can easily be specified a priori.

We illustrate our approach using both simulated data and a real dataset that contains sales data for more than 20 markets each with a different, large, number of products and brands. The remainder of the chapter is structured as follows. In the next section we discuss the model. Next we present the Bayesian inference (some technical details are discussed in the appendix). Section 4.4 shows the results of a simulation experiment. In Section 4.5 we show detailed results of the application of the model to actual data. We conclude the chapter with a discussion.

4.2 Augmented Bayesian BLP Model

In this section we present our approach and we discuss how we augment the BBLP model in the directions discussed earlier. First in subsection 4.2.1 we present the model specification. Next in subsection 4.2.2 we discuss the share inversion method and the integration of the share function.

4.2.1 Model Specification

Consider consumers who make purchases from a set of J products during T time periods in M different markets. In general not all products will be available in all markets. We will use the letter J to refer to the total number of unique products available across all markets. $\mathcal{J}^m$ denotes the set of products that are available in market m. The size of this set, that is, the number of products available in market m is denoted by J^m. In each period a consumer in market m can either choose to purchase one of the products in $\mathcal{J}^m$ or choose an outside good, that is, he buys a product outside the set $\mathcal{J}^m$.

The purchase behavior of individual i in market m is based on utility maximization. We assume that the (latent) utility for consumer i for product j at time t in market m (denoted by u_{ijt}^m) contains three parts, (i) an "explained" part (w_{ijt}^m), (ii) a market level aggregate demand shock (η_{jt}^m), and (iii) an individual level random effect (ϵ_{ijt}^m), that is, we specify

$$u_{ijt}^m = w_{ijt}^m + \eta_{jt}^m + \epsilon_{ijt}^m, \qquad j \in \mathcal{J}^m, t = 1, \ldots, T. \tag{4.1}$$

We make the standard assumption of a type-I extreme value distribution for ϵ_{ijt}^m and we assume $\eta_{jt}^m \sim N(0, \tau_m^2)$. We use a factor structure to further model w_{ijt}^m, that is we use

$$w_{ijt}^m = f_{it}^{m\prime} \lambda_{jt}^m, \qquad j \in \mathcal{J}^m, t = 1, \ldots, T, \tag{4.2}$$

where f_{it}^m denotes an individual-specific K^m dimensional dynamic factor, and λ_{jt}^m is a $(K^m \times 1)$ vector containing the factor loadings for product j in market m. The factor loadings are based on observable product characteristics, such as, packaging and brand name, but also (log) price and promotional indicators may be part of the factor loading vector. In general λ_{jt}^m will contain constant as well as time-varying elements. In principle the same factors will be used in all markets, however, in some cases some factors may not be present in a market. For example, a particular package may not yet be available in a market. Therefore we need to specify the number factors to be dependent on the market.

The factor f_{it}^m gives the importance of a particular product characteristic for individual i in market m at time t. We split this factor into a time-varying part, which is the same across the population, and a heterogeneous part, which is constant over time, that is,

$$f_{it}^m = \bar{f}_t^m + \upsilon_i^m, \text{ where } \upsilon_i^m \sim \phi(0, A^m \Psi A^{m\prime}), \tag{4.3}$$

where Ψ denotes the variance matrix of all individual level random effects and A^m denotes a selection matrix. This matrix selects the rows and columns of the variance matrix that correspond to factors that are relevant for market m. The matrix A^m can be obtained by deleting all rows from the K dimensional identity matrix that correspond to irrelevant factors.

Note that the variance of the random effects is in principle common across markets. Together with the factor loadings in λ_{jt}^m the covariance matrix $A^m \Psi A^{m\prime}$ gives a flexible but parsimonious specification for the variance structure of the preference heterogeneity.

Note that we can write the covariance matrix of the utilities for all products in market m, call this matrix Σ^m, as a function of Ψ, the selection matrices A^m and the factor loadings Λ_t^m where $\Lambda_t^m = \{\lambda_{jt}^m\}_{j \in \mathcal{J}^m}$. That is,

$$\Sigma^m = \Lambda_t^{m\prime} A^{m\prime} \Psi A^m \Lambda_t^m. \tag{4.4}$$

Next we assume a particular law of motion for $\bar{f}_t^m$, the common dynamic component of the factor. We use the state space specification

$$\bar{f}_{t+1}^m = \Gamma_t^m \bar{f}_t^m + \Pi_t^m \omega_t^m, \tag{4.5}$$

where $\omega_t^m \sim N(0, \Omega^m)$ and Γ_t^m is a known matrix. In the state space literature, Ω^m and Γ_t^m are usually set to be diagonal. Furthermore, if we additionally restrict the k-th diagonal element of Γ_t^m to be 1, we obtain a random walk for the k-th factor, that is, $\bar{f}_{kt+1}^m = \bar{f}_{kt}^m + \omega_{kt}^m$. If the variance of ω_{kt}^m is set to zero (or the corresponding element of Π_t^m), we obtain a constant specification for the factor, $\bar{f}_{kt}^m = \bar{f}_{k1}^m$. If we instead set the diagonal element of Γ_t^m to zero and the corresponding variance to a non-zero value, we obtain independent random effects over time, $\bar{f}_{k,t+1}^m = \omega_{kt}^m$.

We complete the model by normalizing the utility of the outside good to be 0. Based on the complete utility specification we can derive the purchase probabilities, or consumption share for individual i, s_{ijt}^m as a function of $(f_{it}^{m\prime} \Lambda_t^m, \eta_t^m)$, where η_t^m is a vector with elements $\{\eta_{jt}^m\}_{j \in \mathcal{J}^m}$ and Λ_t^m is a vector with elements $\{\lambda_{jt}^m\}_{j \in \mathcal{J}^m}$. We use $\{x_{jt}^m\}_{j \in \mathcal{J}^m}$ to refer to a vector containing the elements $(x_{1t}^m, \ldots, x_{Jt}^m)$ and we use $j \in \mathcal{J}^m$ to denote that the product index j is market-specific and hence it covers only the products in the set $\mathcal{J}^m$. Using the properties of the extreme value distribution we obtain

$$s_{ijt}^m(f_{it}^{m\prime} \Lambda_t^m, \eta_t^m) = \frac{\exp(f_{it}^{m\prime} \lambda_{jt}^m + \eta_{jt}^m)}{1 + \sum_{h \in \mathcal{J}^m} \exp(f_{it}^{m\prime} \lambda_{ht}^m + \eta_{ht}^m)}. \tag{4.6}$$

The overall market share, denoted by s^m_{jt}, of product j and time t in market m, measured over the entire population, is obtained by integrating $s^m_{ijt}(f^{m\prime}_{it}\Lambda^m_t, \eta^m_t)$ over the individual-specific parameters in f^m_{it}. Therefore we have that

$$s^m_{jt} = \int \frac{\exp(f^{m\prime}_{it}\lambda^m_{jt} + \eta^m_{jt})}{1 + \sum_{h\in\mathcal{J}^m}\exp(f^{m\prime}_{it}\lambda^m_{ht} + \eta^m_{ht})}\phi(f^m_{it}; \bar{f}^m_t, A^m\Psi A^{m\prime})df^m_{it} \tag{4.7}$$

If we use $f^m_{it} = \bar{f}^m_t + \upsilon^m_i$ and $\upsilon^m_i \sim \phi(0, A^m\Psi A^{m\prime})$ we can write equation (4.7) as

$$s^m_{jt} = \int \frac{\exp(\mu^m_{jt} + \lambda^{m\prime}_{jt}\upsilon^m_i)}{1 + \sum_{h\in\mathcal{J}^m}\exp(\mu^m_{ht} + \lambda^{m\prime}_{ht}\upsilon^m_i)}\phi(\upsilon^m_i; 0, A^m\Psi A^{m\prime})d\upsilon^m_i, \tag{4.8}$$

where $\mu^m_{jt} = (\bar{f}^m_t)'\lambda^m_{jt} + \eta^m_{jt}$. Note that the share s^m_{jt} inherits randomness only from the term η^m_{jt} as we integrate over υ^m_i.

Following Jiang et al. (2009) we denote the relationship between the shares vector $s^m_t = \{s^m_{jt}\}_{j\in\mathcal{J}^m}$ and the vector with aggregate demand shocks η^m_t in (4.7) as

$$s^m_t = h(\eta^m_t | \Lambda^m_t, \bar{f}^m_t, \Psi). \tag{4.9}$$

Based on the relation in (4.9) and the distribution of η^m_t, the joint density of the shares at time t is

$$\pi(s^m_t | \Lambda^m_t, \bar{f}^m_t, \Psi, \tau^2_m) = \phi(h^{-1}(s^m_t | \Lambda^m_t, \bar{f}^m_t, \Psi) | 0, \tau^2_m) |J_{s^m_t \to \eta^m_t}|^{-1}, \tag{4.10}$$

for $t = 0, \ldots, T$ and $m = 1, \ldots, M$ and where we use $\pi(.)$ to denote a generic density and $\pi(y|x)$ the density of y given x. In addition, the Jacobian $J_{s^m_t \to \eta^m_t}$ is defined as the $(J^m \times J^m)$ matrix with elements

$$\partial s^m_{jt}/\partial \eta^m_{kt} = \begin{cases} -\int s^m_{ijt}s^m_{ikt}\phi(\upsilon^m_i; 0, A^m\Psi A'_m)d\upsilon^m_i & \text{if } k \neq j \\ \int s^m_{ijt}(1 - s^m_{ikt})\phi(\upsilon^m_i; 0, A^m\Psi A'_m)d\upsilon^m_i & \text{if } k = j, \end{cases} \tag{4.11}$$

where the arguments of the functions s^m_{ijt} and s^m_{ikt} are dropped for convenience, see (4.8) and $j, k \in \mathcal{J}^m$. Given equation (4.10) the joint conditional density for the shares, or the likelihood, for market m is given by

$$\pi(s^m|\Lambda^m, \bar{f}^m, \Psi, \tau_m^2) = \prod_{t=1}^{T} \pi(s_t^m|\lambda_t^m, \bar{f}_t^m, \Psi, \tau_m^2), \tag{4.12}$$

where $\Lambda^m = (\Lambda_1^m, \ldots, \Lambda_T^m)$, $s^m = (s_1^m, \ldots, s_T^m)$ and $\bar{f}^m = (\bar{f}_1^m, \ldots, \bar{f}_T^m)$.

Two difficulties in this model are the inversion of the share function $h()$ in equation (4.9) and the evaluation of the integrals in equations (4.8) and (4.11). The inversion and the integration are required to obtain the aggregate shocks η_t^m and hence to evaluate the density in equation (4.10). We discuss these two issues next.

4.2.2 Share inversion method and integral approximation

To calculate the joint density in (4.12) we need to take two hurdles. First we need to solve the integrals in (4.8) and (4.11). Next, we need to obtain the inverse of the function $h()$ in (4.9).

We apply the contraction mapping of Berry et al. (1995) to obtain the inverse in terms of μ_{jt}^m for all necessary m, j and t. Within this procedure we need to calculate the market shares given μ_{jt}^m, Λ_t^m and Ψ, $j \in \mathcal{J}^m$, $t = 1, \ldots, T$ and $m = 1, \ldots, M$ by integrating equation (4.8) with respect to υ_i. We numerically approximate this integral by averaging over H draws from the distribution of υ_i that is $N(0, A^m \Psi A^{m\prime})$. Jiang et al. (2009) report that H ranges from 20 to 50 in previous literature and they show that their Bayesian estimator has the same performance for $H = 50$ and $H = 200$. However, in our case we may need more draws as we develop the model for many more parameters.

A common approach to obtain each of the H draws of υ_i is based on the product of the Cholesky decomposition of $A^m \Psi A^{m\prime}$ and draws from a standard normal, that is $\upsilon_i^d = (A^m \Psi A^{m\prime})^{1/2} \zeta^d$ where $\Sigma^{1/2}$ denotes the Cholesky decomposition of Σ and $\zeta^d \sim N(0, \mathbf{I})$ for $d = 1, \ldots, H$, where $\mathbf{I}$ denotes an identity matrix. A more efficient approximation of the integral may be obtained by using a quasi-random scheme to generate the ζ^d. Train (2003, chap. 9, page 236) suggests scrambled Halton sequences for integrals of large dimensions and his suggestion, we believe, is motivated by the same family of logit models that we are concerned with here.

In Figure 4.1 we compare the integration results based on scrambled Halton sequences versus the integration results based on regular normal draws. We consider the scenario where the parameters are known and we use the approximation method discussed above to obtain the market shares. In the top panel we report the performance when the integral has only three dimensions and in the lower panel we report the performance when the integral has 30 dimensions. In both panels we report the market share for only one of the products. This simple exercise suggests that the market shares are much better approximated by integrating with Halton draws regardless of the dimension of the integral. For large dimensions the approximation of the normal draws seems to converge to the approximation of the Halton draws after the number of draws (H) is higher than 400 while the approximation based on Halton draws performs well for $H > 100$.

4.3 Bayesian Inference

In this section we discuss the priors we choose to complete the model specification. Specifically, we present in subsection 4.3.1 a prior for the matrix Ψ that is simple to calibrate when analyzing many products and markets and at the same time the prior will let us treat the scale and the correlation structure of Ψ separately. Next in subsection 4.3.2, we discuss the market-specific priors. Finally in subsection 4.3.3, we discuss the MCMC sampling scheme.

4.3.1 Prior and Structure for Ψ

Jiang et al. (2009) specify the covariance matrix Σ^m in terms of the unique elements of its Cholesky root. They set $\Sigma^m = U'U$ where

$$U = \begin{pmatrix} e^{r_{11}} & r_{12} & r_{13} & \dots & r_{1J} \\ 0 & e^{r_{22}} & r_{23} & \dots & r_{2J} \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & e^{e_{J-1,J-1}} & r_{J-1,J} \\ 0 & 0 & \dots & 0 & e^{r_{JJ}} \end{pmatrix}, \tag{4.13}$$

and they choose to set separate normal priors for the diagonal and off-diagonal elements of U. That is, Jiang et al. (2009) set $r_{jj} \sim N(0, \sigma_j^2)$ for the diagonal elements $j = 1, \dots, J$ and $r_{jk} \sim N(0, \sigma_{od}^2)$ for the off-diagonal elements $j \neq k$. Note that Jiang et al. (2009) deal with only one market m and that in our approach we model the heterogeneity through Ψ (that is a $K \times K$ matrix) and not through Σ^m (that is a $J \times J$ matrix) where K is the number of factors while J is the number of products.

This prior specification enforces the positive-definitiveness of Σ^m and in addition the priors on the r_{jk} elements used by Jiang et al. (2009) are symmetric and this matched well with the random walk Metropolis Hastings [MH] sampling scheme they propose to sample the r-parameters. A second advantage of Jiang et al. (2009) prior is that it can be set to be relatively uniform on the correlation range $(-1, 1)$. Overall, this specification is attractive and simple but it also has a number of shortcomings. First, Jiang et al. (2009) note that to obtain a plausible (implied) prior on the variances in Σ^m the prior variances σ_j^2 should be decreasing with j and they provide a particular relation between σ_j^2 and j. However, the prior in one of the elements of (4.13) may affect many of the elements in Σ^m and this complicates the prior interpretation, specially when J is large. Second, this prior imposes a correlation structure simultaneously with the overall scale of the heterogeneity. Other studies point out that it may be relatively hard to identify the heterogeneity (Bodapati and Gupta, 2004) and the uncertainty related to the Σ^m elements is therefore usually large, see for example Jiang et al. (2009) and Musalem et al. (2006). However, we do not know if the large uncertainty reported in previous studies is due to the uncertainty on the overall scale of Σ^m or if it is due to the correlation structure in Σ^m. Finally, it is well known that the correlation structure of Σ^m is very important in order to obtain different substitution patterns far from the IIA assumption of the logit. Therefore, we would like to use a prior that can let us deal with the scale and correlations separately.

Finally, changing one element of U may lead to a very different Σ^m. This fact makes the implementation of an efficient MH sampler difficult if J is large. More precisely, in their MH scheme Jiang et al. (2009) choose to draw the candidate elements of U from a multivariate normal that is calibrated based on a short chain of their model MCMC output. The length of the chain needed for the calibration of the multivariate normal needs to be longer when the number of dimensions is large. When dealing with large dimensions, the step size in the random walk MH sampling, for each of the r_{jk} elements, needs to be set smaller as the dimension increases in order to keep a good acceptance rate in the estimation algorithm. Although this last issue always arises whenever many elements are updated simultaneously, here it can be more dramatic as "local" changes in U lead to "global" changes in Σ^m.

Summarizing, we would like to use a prior specification that is simpler to calibrate when dealing with large dimensions and at the same time we like to treat the scale and the correlation structure of the heterogeneity separately.

We choose to use the prior specification of Barnard et al. (2000) for Ψ. We define $\Psi = DRD$ where D is a diagonal matrix with K elements (standard deviations) and R is a $K \times K$ correlation matrix. For the variances in D we set the prior $\log(\text{diag}(D)) \sim N(0, \Delta)$. Formulating a prior on R is not straightforward because we need a prior that deals with all the elements of R and the restrictions on them. We need to assure the positive definitiveness of R, the range of its elements must be $(-1, 1)$ and all the elements together should satisfy triangularity restriction inherent to any correlation matrix. In addition, we need to update all the elements of R simultaneously to ease the computational burden. However, based on any variance matrix Σ one can obtain the corresponding correlation matrix by standardization. Hence, we assume $R = f_c(S)$ and we specify an Inverted Wishart prior for S with parameters (G, v). The function $f_c()$ transforms a covariance matrix to a correlation matrix. The location matrix G is set such that the expected value of S is an identity matrix; that is $G = (v - K - 1) \times \mathbf{I}$, v is the degrees of freedom of the Inverted Wishart and $\mathbf{I}$ is an identity matrix of size K. Note that our variance matrix Ψ is now actually a function of D and S. In the MCMC sampling below we will actually sample these two matrices.

Barnard et al. (2000) set a prior directly on R while we set a prior on R implicitly by the prior on S. The main reason why we deviate from them is that evaluation of the posterior is very costly in our application and hence we need to use a proposal for S that updates all the correlations in R in a single step. In contrast, the computation time of the application in Barnard et al. (2000) allows for a relatively fast element by element update of the matrix R.

In Figure 4.2 we report the implied correlation distributions for two different degrees of freedom and for two elements of a Ψ matrix of size 10×10. The implied correlations can be set to be roughly uniform on the $(-1, 1)$ interval depending on the degrees of freedom set on the Wishart distribution. Therefore, the implied correlations of this prior are very similar to the implied correlations of the specification used by Jiang et al. (2009). However, the parameters in our suggested priors are easier to interpret.

4.3.2 Market-Specific Priors and Joint Posterior

We presented the likelihood for each of the m markets in equation (4.12) and we presented the structure and prior for Ψ in the previous section. This variance matrix applies to all markets. What is left to specify are the priors for τ_m^2, Ω^m and the initial state distributions for all common dynamic factors f_t^m. In addition the matrix Γ_t^m needs to be defined. We define $f_1^m \sim N(0, P^m)$ where we set P^m reasonably large and non-informative, $P^m = 100\mathbf{I}$ for all m. We assume $\Omega^m = \sigma_m^2 \mathbf{H}_m$ where $\mathbf{H}_m$ is a diagonal matrix of size J^m and $\sigma_m^2 \sim v_o s_o^2 / \chi_{v_o}^2$. The diagonal elements of $\mathbf{H}_m$ are equal to one for the factors f_t^m that are time-varying and equal to zero for the factors that are fixed over time. We set Γ_t^m equal to an identity matrix $\mathbf{I}_m$ of size J^m and for τ_m^2 we do not set any prior.

The joint posterior is proportional to the product of the likelihood and priors for each market times the prior distribution of Ψ that apply to all markets. Note that the factor loadings are assumed to be given, as they represent observed product characteristics. The posterior becomes

$$\begin{aligned}\pi(\bar{f}^*, \tau_*^2, \sigma_*^2, D, S | s^*, \Lambda) \propto\ & \pi(\log(\mathrm{diag}(D)); 0, \Delta)\pi(S; G, v) \times \\ & \left(\prod_m \pi(s^m | \bar{f}^m, \Psi, \tau_m^2) \pi_m(\bar{f}_1^m; 0, P^m) \left[\prod_{t=1}^{T-1} \pi(\bar{f}_{t+1}^m | \bar{f}_t^m, \sigma_m^2 \mathbf{I}_m) \right] \pi(\sigma_m^2; v_o, s_o^2) \right),\end{aligned} \tag{4.14}$$

where $s^* = (s_1, \ldots, s_M)$, $\bar{f}^* = (\bar{f}^1, \ldots, \bar{f}^M)$, $\tau_*^2 = (\tau_1^2, \ldots, \tau_M^2)$, $\sigma_*^2 = (\sigma_1^2, \ldots, \sigma_M^2)$, $\Psi = Df_c(S)D$.

In addition, the priors for f_1^m, σ_m^2, D and S are defined as follows

$$\begin{aligned}
&\pi(\bar{f}_1^m; 0, P^m) \sim N(\bar{f}_1^m; 0, P^m) \\
&\pi(\sigma_m^2; v_o, s_o^2) = (\sigma^2)^{-(v_o/2+1)} e^{-v_o s_o^2/2\sigma^2} \\
&\log(\text{diag}(D)) \sim N(0, \Delta) \\
&\pi(S; G, v) \propto \tfrac{|G|^{v/2}}{|S|^{(v+K+1)/2}} e^{-1/2tr(S^{-1}G)}
\end{aligned} . \tag{4.15}$$

4.3.3 MCMC algorithm

The approach we follow is a combination of the sampler proposed in Jiang et al. (2009) with simulation smoother of Durbin and Koopman (2002) and a Metropolis Hastings sampler for Ψ. We use the following steps: (i) conditional on Ψ we use the contraction mapping to obtain the (implied) μ_{jt}^m, for $m = 1, \ldots, M$, $j \in \mathcal{J}^m$, $t = 1, \ldots, T$; (ii) conditional on Ψ (and μ_{jt}^m) we use the simulation smoother to sample $\bar{f}^*$, the μ_{jt}^m values appear as dependent variables in this smoother; (iii) conditional on $\bar{f}^*$ and all μ_{jt}^m we sample τ_*^2 and σ_*^2; (iv) finally we use a Metropolis Hastings sampler to draw the elements of D and S which determine Ψ.

More specifically, we use the following three set of conditionals

$$\begin{aligned}
&\bar{f}^* | \Psi, \sigma_*^2, \tau_*^2, s^*, \Lambda \\
&\sigma_*^2, \tau_*^2 | \Psi, \bar{f}^*, s^*, \Lambda \\
&D, S | \sigma_*^2, \tau_*^2, \bar{f}^*, s^*, \Lambda.
\end{aligned} \tag{4.16}$$

We draw the first set of conditionals using the simulation smoother of Durbin and Koopman (2002). That is, given μ_{jt}^m for all m, j and t we can draw the parameters of the following measurement and state equations

$$\begin{aligned}
\mu_t^m &= \Lambda_t^{m\prime} \bar{f}_t^m + \eta_t^m \qquad \text{with} \quad \eta_{jt}^m \sim N(0, \tau_m^2) \\
\bar{f}_{t+1}^m &= \Gamma_t^m \bar{f}_t^m + \Pi_t^m \omega_t^m \quad \text{with} \quad \omega_t^m \sim N(0, \sigma_m^2 \mathbf{I}_m),
\end{aligned} \tag{4.17}$$

where μ_t^m is defined as the $(J_m \times 1)$ vector with elements μ_{jt}^m, $j \in \mathcal{J}^m$. This specification is attractive because we can set some of the common factors $\bar{f}_t^m$ to be fixed in time while others can remain time-varying. This is done simply by setting some of the elements in the diagonal matrix

Π_t^m equal to zero. The simulation smoother of Durbin and Koopman (2002) gives a draw from the joint posterior of $\bar{f}_t^m$, for $t = 1, \ldots, T$. For details we refer to their paper. Conditional on μ_t^m and $\bar{f}^m$ sampling the variances is straightforward. Given our priors they can be sampled from Inverted Gamma distributions. That is, $\tau_m^2 \sim IG(n_\tau^m, s_\tau^2)$ and $\sigma_m \sim IG((v_o + n_\sigma^m), (s_\sigma^2 + s_o^2)$. The n_τ^m are the number of observations available for the measurement equation at market m for $m = 1, \ldots, M$ and s_τ^2 is the sum of squared residuals of the measurement equation. The n_σ^m is the number of observations available in the state equation at market m and s_σ^2 is the sum of squared residuals of the measurement equation. The v_o and s_o^2 are the parameters of the prior for the variance of the state equation, see the priors in equation (4.15).

For the third set of conditionals we use a Metropolis Hastings algorithm. For the (log of the) elements of D we use a standard random walk as candidate distribution. For comparison with the second part of this step we write the proposal as

$$\log(\mathrm{diag}(D^{\mathrm{candidate}})) \sim N(\log(\mathrm{diag}(D^{\mathrm{current}})), \zeta^2 \mathbf{I}). \tag{4.18}$$

For S we also propose a random walk candidate distribution. However, for efficiency in the total sampler we which to have a candidate that can generate matrices close to the current value. As a candidate distribution we use an inverted Wishart distribution which has the current value as expected value, that is,

$$S^{\mathrm{candidate}} \sim IW((v_1 - K - 1)S^{\mathrm{current}}, v_1). \tag{4.19}$$

We choose v_1 and ζ^2 to achieve between 20% and 50% acceptance rate in the Metropolis steps. In the MCMC we use two Metropolis steps to update D and S separately.

To sample D and S we evaluate the model posterior in equation (4.14) in two Metropolis steps, the first for D and the second for S. We set

$$D^{new} = D^{cand} \quad \text{with probability} \quad \min\{\tfrac{p^*(D^{cand}|S,\bar{f}^*,\tau_*^2,\sigma_*^2,s^*,\Lambda)}{p^*(D^{prev}|S,\bar{f}^*,\tau_*^2,\sigma_*^2,s^*,\Lambda)}, 1\}, \tag{4.20}$$

and we set

$$S^{new} = S^{cand} \quad \text{with probability} \quad \min\{\tfrac{p^*(S^{cand}|D,\bar{f}^*,\tau_*^2,\sigma_*^2,s^*,\Lambda)}{p^*(S^{prev}|D,\bar{f}^*,\tau_*^2,\sigma_*^2,s^*,\Lambda)}, 1\}. \tag{4.21}$$

The candidate and previous posterior density in equation (4.20) are given by

$$p^*(D^{cand}|S, \bar{f}^*, \tau_*^2, \sigma_*^2, s^*, \Lambda) = \\ \pi(\log(\mathrm{diag}(D^{cand})); 0, \Delta) \times \prod_m \pi(s^m|\Lambda^m, \bar{f}^m, \Psi^{cand}, \tau_m^2), \tag{4.22}$$

and by

$$p^*(D^{prev}|S, \bar{f}^*, \tau_*^2, \sigma_*^2, s^*, \Lambda) = \\ \pi(\log(\mathrm{diag}(D^{prev})); 0, \Delta) \times \prod_m \pi(s^m|\Lambda^m, \bar{f}^m, \Psi^{prev}, \tau_m^2). \tag{4.23}$$

where $\Psi^{cand} = D^{cand} f_c(S) D^{cand}$ and $\Psi^{prev} = D^{prev} f_c(S) D^{prev}$ while the terms in the Metropolis step in equation (4.21) are given by

$$p^*(S^{cand}|D, \bar{f}^*, \tau_*^2, \sigma_*^2, s^*, \Lambda) = \\ \pi(S^{cand}; G, v) \times \pi(S^{prev}; S^{cand}, v_1) \times \prod_m \pi(s^m|\Lambda^m, \bar{f}^m, \Psi^{cand}, \tau_m^2), \tag{4.24}$$

and by

$$p^*(S^{prev}|D, \bar{f}^*, \tau_*^2, \sigma_*^2, s^*, \Lambda) = \\ \pi(S^{prev}; G, v) \times \pi(S^{cand}; S^{prev}, v_1) \times \prod_m \pi(s^m|\Lambda^m, \bar{f}^m, \Psi^{prev}, \tau_m^2), \tag{4.25}$$

where now $\Psi^{cand} = D f_c(S^{cand}) D$, $\Psi^{prev} = D f_c(S^{prev}) D$ and $\pi(s^m|\Lambda^m, \bar{f}^m, \Psi, \tau_m^2)$ is defined in equation (4.12). Note that the proposal distribution for S is not symmetric and hence its distribution ($\pi(S; S', v_1)$) also appears in the acceptance probability. We use the proposal distributions in equation (4.18) and equation (4.19) to draw the candidate matrices D^{cand} and S^{cand} based on their previous values D^{prev} and S^{prev}, respectively.

The Metropolis steps are very costly in terms of computation time in the MCMC algorithm. This is the only step in the algorithm where we need to use the BLP contraction mapping and where we need to evaluate the Jacobian in equations (4.10) and (4.11). Some time may be saved by jointly updating these matrices. However, the joint updating of D and S will not let us distinguish what is driving the acceptance rate in the Metropolis steps. Moreover, the separate updating of these matrices let us distinguish if a candidate matrix Ψ^{cand} ($\Psi = Df_c(S)D$) is rejected because of its correlation structure or because of its overall scale.

4.4 Simulation Experiment

We test our modeling approach on simulated data and in this section we discuss the data generation process and the results of the MCMC estimation procedure.

4.4.1 Data Simulation

In this section we describe how we create synthetic data and we consider a setting where we have data for many products and markets. This setting is not typical in the literature but it that corresponds with the setting that we deal with in the application.

We assume products are sold in 10 markets and we simulate 4 years of monthly data for each market. Each market will be assigned a specific number of products and these products will be assigned to 10 different brands.

All 10 brands are available in each market and we assign 5, 6, 8 or 10 products to each brand at each market. Hence, the number of products assigned to a brand varies per market and each market consists of a specific number of products. The probability of a brand to be assigned 5 or 6 products at each specific market is 90% while the probability of being assigned 8 or 10 products is 10%. That is, the expected number of products per brand is 5.85 and the expected number of products per market is 58.5. This is a large number of products relative to previous studies. For example, Jiang et al. (2009) and Yang et al. (2003) study one market that consists of 3 products and one outside good while Musalem et al. (2006) apply their model to a setting with four products and one outside good.

The mean utilities (the μ_{jt}^{m}) for products are market-specific. The mean products' utilities are assumed to depend on the products' brands, the products' attributes and the products' prices and promotions. We define 5 attributes and we assign only one attribute per product and all attributes are equally likely per product. Note that this last assumption implies that each brand may have a certain number of products that share the same attributes. We set one of the attribute coefficients as the base and equal to 0 while the rest is generated from a normal distribution with mean 0 and variance 5.

Next, we define the price and promotion coefficients and these are -5 and -2, respectively, and these coefficients are the same across markets. The price series for each product follows a sine curve (with a very long cycle) plus normal noise with mean 2 and variance 1. To create the promotional series we use a uniform distribution with range $(0, 1)$. We assume that there is a 30% chance of a promotion and the range of promotions is between 0 and 30%. That is, when we draw a promotional index value (from the uniform) lower than 0.70 then the promotional index is equal to 1 otherwise the promotion index is equal to the drawn value.

Attribute, price and promotions coefficients will be fixed in time while the brand coefficients will be time-varying. We generate 10 brand coefficients using the recursion in equation (4.17) and we set σ_m to be equal to 0.40. We set the initial values for the brand coefficients f_1^m based on a normal distribution with mean -3 and variance 0.16. We use the same recursion to generate the attribute, price and promotion coefficients and their initial value is assigned as we discussed in the previous paragraph. We further need to set Π_t^m to be a diagonal matrix with the first 10 elements equal to 1 and the remainder 6 elements of the diagonal are equal to 0. The Γ_t^m is equal to an identity matrix of size 16.

The factor loadings Λ_t^m will consist of brand and attribute dummies for all products at time t plus the products prices and promotions at time t. That is, Λ_t^m is a $J \times K$ matrix, J is the number of products and K is equal to 16 (the number of brands, attribute, price and promotion coefficients). Finally, we assume that the variance of product demand shocks τ_m^2 are equal to 0.8 for all m.

We use the specification of $\Psi = DRD$ to draw the random coefficients υ_i^m. We first draw a matrix P based on a $IW(I_{16}, 21)$ and we set $R = f_c(P)$. The implied range of the correlations in R goes from -1 to 1 but the extremes of the range are not common. Further, we assume the scale of the heterogeneity depends on both small and large elements with the purpose of checking whether their size affects their retrieval from the synthetic data. That is, we set $D^2 = (2, 2, 2, 2, 8, 8, 8, 8, 4, 4, 4, 4, 2, 2, 2, 2)$. Finally, we use 3000 draws to approximate the integral in equation (4.8) and we generate the draws of the random coefficients based on the Cholesky decomposition of Ψ and normal draws generated with scrambled Halton sequences. The 16 factors in Ψ are available at every market and therefore the A^m matrix is the same for all markets and it is equal to I_{16}.

4.4.2 MCMC Setup

We use a hybrid Metropolis Gibbs sampler to estimate the parameters of the model in equation (4.7). The sampler iterates over the conditionals in equation (4.16). The first set of conditionals concerns the $\bar{f}^*$. We set the prior on the initial values as $f_1^m \sim N(0, 100\mathbf{I})$ for all m and we use the simulation smoother of Durbin and Koopman (2002) to sample all elements of $\bar{f}^*$.

The second set of conditionals samples the variances of equation (4.17). We did not set any prior information on σ_m^2 and τ_m^2 for all m. Hence, $\tau_m^2 \sim IG(n_m, s_m)$ where n_m are the number of observations in the measurement equation in (4.17) and s_m are the sum of squared residuals in the same equation. In a similar fashion, $\sigma_m^2 \sim IG(n_m^s, s_m^s)$ where n_m^s are the number of observations and s_m^s are the sum of squared residuals of the state equation in (4.17).

The third set of conditionals concerns the sampling of the D and S matrices. We set the υ parameter in the prior $\pi(S; \mathbf{I}, \upsilon)$ equal to 21 and we use $\Delta = 10\mathbf{I}$ in the normal prior of the log of the diagonal elements of D.

We use the proposal distributions in equation (4.18) and equation (4.19) to draw the candidate matrices D^{cand} and S^{cand}, respectively. In these proposals we set ζ^2 equal to 0.01 and υ_1 equal to 10000. The large number in υ_1 corresponds to steps of approximately 0.05 in the elements of the correlation matrix R where $R = f_c(S)$. We calibrated ζ^2 and υ_1 to achieve an acceptance rate between 20% and 50% for both Metropolis steps.

We let the Gibbs-Metropolis sampler to run for 20 thousand iterations. However, we do oversampling of Ψ. We use 4 updates of S and one of D at every iteration. That is, we generate 100 thousand candidates for the matrix Ψ. The matrix Ψ contains 136 unique elements and our purpose with the oversampling is to let all these elements to move at larger steps at every iteration and let them adjust better to the rest of the model parameters drawn at every iteration. In this way, the oversampling may compensate for the small moving steps that we need to achieve a good acceptance rate in the Metropolis algorithm. Haran et al. (2003) also consider the oversampling of parameters to accelerate their computation in the MCMC algorithm.

4.4.3 Results of the Simulation Experiment

In Table 4.1 we present the posterior mean and the 99% Highest Posterior Density Region (HPDR) of the demand shocks for every market. The true value of τ_m^2 is equal to 0.66 for all markets. Note that we generated data for 10 markets. In most cases, the posterior mean is very close to its true value. The maximum absolute deviation of the posterior mean from the true value is approximately 0.06, see the $\tau_{m=6}^2$ that is equal to 0.580.

In Table 4.2 we present the posterior mean and HPDR of the variance term in the state equation (4.17), that is σ_m^2. The true value of this parameter is 0.16 while in most cases the posterior mean is close to 0.12. That is, we are finding a small negative bias that is close to 0.04 for most cases.

In Figure 4.3 we present the estimates of the fixed coefficients (in circles) and the box-plots of their posterior distribution. Note that we specified 4 attribute coefficients and one price and promotion coefficient that vary across markets. That is a total of 50 coefficients in all markets. We see that for 30 out of the 50 coefficients the circles (true values) overlap with the position of their distribution in the box-plot. In the same figure, we see that there is a systematic positive bias in the posterior distribution of the price coefficients. The true value of the price coefficient is equal to -5 while the posterior distribution is higher than -5. In contrast, the posterior distribution of the promotion coefficients overlap with its true value (-2) for all markets.

In Figure 4.4 we present the distribution of the time-varying brand coefficients for the 5^{th} market. We see that the overall time profile is well retrieved by the estimation algorithm. In most cases the true value is inside the 99% HPDR. The results for the other markets are very similar.

In Table 4.3 we report the posterior mean and HPDR for the elements of the D^2 matrix. We see that the 99% HPDR contains the true value for 7 out of the 16 elements. The deviation of the posterior mean from its true value, when the true value is not contained in the HPDR, may be as small as 1 or as large as 6 variance points. That is, we find large uncertainty regarding the scale of the heterogeneity driven by the random coefficients. Jiang et al. (2009), Musalem et al. (2006) and Yang et al. (2003) find similar levels of uncertainty.

In Figure 4.5 we report the 99% HPDR (in dashed lines) and the true value (solid line) of the 120 unique elements of the correlation matrix R $(f_c(S))$. We find that the HPDR contains the true value for 57 out of 120 elements, that is 47.5% of the elements. However, we find that the posterior mean of the correlations is on average 0.16 points far from its true level. Hence, our results suggest that the uncertainty regarding the scale of the heterogeneity (the elements of D) is much larger than the uncertainty in the elements of the correlation matrix R.

4.5 Empirical Application

In this section we apply our estimation approach to a real dataset and we analyze the substitution patterns between a large number of products. Next we provide a description of the data (in subsection 4.5.1), the modeling details (in subsection 4.5.1) and the estimation results (in subsection 4.5.3).

4.5.1 Data

Our dataset contains sales, price and promotion data for all the products of one supermarket food category. The data is monthly and it covers a period of four years and 18 different regions. Consumers at each region may have available a minimum of 25 up to a maximum of 65 products of 20 different brands. Each brand has its own positioning in terms of calories, taste and labeling while each brand may offer products of the same size and packaging. Therefore, we can describe each product in terms of its brand, size and packaging attributes and its price and promotion data. There are brands with similar attributes both in terms of calories and taste and in terms of packaging and size and these brands are usually produced by different companies. Our data contains products sold by all major companies at each market and very few firms compose the market.[1] Depending on the market, the size of the outside good varies from 20 up to a maximum of 50%. The calculation of the outside good share is region-specific and it varies according to the share of the closest and competing food categories.

4.5.2 Modeling Details and MCMC Setup

The MCMC setup for the application is very similar to the MCMC setup we use for the simulated data. An important distinguishing feature is that the matrix Ψ consists of 32 rows and columns. This number corresponds to 20 brands, 11 size and packaging attributes, one price and one promotion factor. We leave one attribute as reference and this results in 32 random coefficients. In the application the A^m matrices select the appropriate elements of the Ψ matrix relevant for the market m. That is, some attributes or brands may not be available in all markets.

We will assume that all coefficients are fixed with the exception of the brand coefficients that will be specified as time-varying. We use the priors in equation (4.15) where we set $P^m = 100\mathbf{I}$ for all m, $v_o = 1$ and $s_o = 0.01$. The Δ matrix is equal to $25\mathbf{I}$ and $v = 35$. We did not set a prior on τ_m^2 parameters. The proposal distributions in equation (4.18) and equation (4.19) have the parameters $v_1 = 30000$ and $\zeta^2 = 1/200$. This configuration achieves between 30% and 50% of acceptance rate in the Metropolis updates of S and D. We sample D and S separately in the same way as we did in the simulation experiment.

The matrix Π_t^m in equation (4.17) is set equal to an identity matrix of size K^m (K^m is the number of factors at each m) and we set some of its diagonal elements equal to 0 and these zeros correspond to the factors related to size, packaging and to price and promotions. The matrix Γ_t^m is of size J^m (the number of products available at market m) times K^m and it is also set to be an identity matrix.

We ran the MCMC chain for 50 thousand iterations and we discarded the first 10 thousand with a thin value of 20. The computation time was of approximately five days. The number of

[1] Because of our confidentiality agreement we can not reveal the companies names, brands or any other product or market information in the chapter.

draws that we used for approximating integrals was 200 and we use draws based on scrambled Halton sequences.

4.5.3 Estimation Results

We present the posterior mean and HPDR of the τ_m^2 parameters in Table 4.4. The uncertainty of the demand shocks is very large for six of the eighteen markets, see the τ_m^2 for m equal to 1, 2, 12, 13 and 17. The uncertainty in the demand shock for the remaining markets seems small relative to these six markets.

The posterior mean of HPDR of the σ_m^2 parameters can be read in Table 4.5. These are the variances of the time-varying coefficients and we see that they are very small as we expected. The variance of time-varying parameters in state space models is usually small (Fruhwirth-Schnatter, 2004) and this indicates slowly evolving factors.

In Table 4.6 we present the posterior mean and HPDR for the fixed coefficients at three markets. We notice that price and promotion coefficients have the expected negative signs. The promotional index is a number that takes a value between 0 and 1 and it indicates the percentage of the regular price level that is observed. We notice that the uncertainty related to the price coefficients varies across markets while at the same time they remain negative. The preference for *size* also varies per market and we find that for each market there are only two *sizes* with a positive posterior mean that may be larger than the base category. In Figure 4.6 we report the evolution of the time-varying brand coefficients. We report the time profiles of the time-varying factors relative to their starting point and their corresponding 99% HPDR. This transformation is useful to illustrate how some brands' preferences (measured by the time-varying factors) face large variations relative to their starting position, like brands C, E, F or L, while we see other brands like J or B with much smaller time variation. Note that this figure does not show the level uncertainty around the time-varying brand coefficients. Their level uncertainty, however, is similar to the uncertainty of the fixed coefficients. We see also that the coefficients for different types of packaging show significant time variation relative to their starting point, see the bottom row in Figure 4.6.

In Figure 4.7 we report the distribution of 60 elements of the Ψ matrix. The Ψ matrix size is 32×32 and therefore it contains 528 unique elements. We notice that the uncertainty varies per element but overall the uncertainty is relative small for most correlations. The element 39 in the lower panel has the largest uncertainty and its range goes from -0.75 up to 0 while there are other cases like the element 12 in the upper panel with very tight posterior distributions.

In Table 4.7 we present the posterior mean and the 99% HPDR of the matrix D^2. Some of the elements of the matrix are retrieved with a lot of uncertainty. For example, the posterior mean of the D_9^2 is 4.64 but its HPDR includes values close to 10 while the posterior mean of D_{15}^2 is equal to 10.625 and its HPDR includes values as high as 22. These rest of the elements in the D^2 matrix, and the mayority, show a much smaller uncertainty relative to these high values in D_9^2 and D_{15}^2. Previous studies, like Jiang et al. (2009), Musalem et al. (2006) and Yang et al. (2003), report simular range of both the scale of the heterogeneity and its uncertainty.

Finally, in Figure 4.8 and Figure 4.9 we present the own-price and cross price elasticities for the products in market 2. We computed the elasticities as we describe in the Appendix 4.A. The price elasticities have a range that goes from -1 to -3.5 while the cross-price elasticities range goes from 0 up to 1.6. In this last figure light (white) colors represent high values while darker (dark red) colors represent lower cross price elasticities. In Figure 4.9 we notice that many products respond to the price changes of a relatively small set of products. For example, a price change in the 10^{th} product affects almost all products in this market and their cross price elasticity is close to 1.66. Finally, we notice that substitution patterns (measured by cross price elasticities) are stronger among a small subset of products.

4.6 Conclusions

The estimation of aggregate share models based on the random coefficient logit specification presents different challenges. The scalability of models and estimation algorithms is one of these main challenges. Berry and Pakes (2007) is a recent paper with a similar concern as ours and that is the practical application of this family of models to larger and more comprehensive datasets. In this chapter we investigate the scalability of the BBLP approach and we successfully applied our method to simulated data and to a relative large real dataset. It is large in terms of the number of products, brands and markets that it includes while it is still small relative to the time periods we have available.

Our specification is based on the recent advances of Jiang et al. (2009), Durbin and Koopman (2002) and Barnard et al. (2000). These advances all put together allow us to model time variation in preferences and to separate the uncertainty of the random coefficients in terms of their scale and in terms of their correlation. In addition, our mode specification combines global and market specific priors and this allows us pool information across several markets.

We believe that the uncertainty related to the random coefficients is a great challenge. In contrast with previous studies we report the uncertainty related to the correlation and the scale of the random coefficients separately. Our results point that the overall scale of the covariance matrix of the random coefficients may present a larger uncertainty relative to the uncertainty present in their correlation structure. This last result is an initial step towards the untangling and modeling of the sources of uncertainty in the random coefficients of the BBLP approach and we consider that this is a promising area for further research.

We present an approach that is the "augmented" version of the BBLP and it should be considered whenever there is a large dataset of market shares available for analysis. Large datasets, particularly of shares, are rarely collected but they are becoming increasingly common and more detailed. Therefore, approaches like ours may be needed more often in the future.

We presented our results to managers and they showed a great interest in understanding the uncertainty regarding the correlations between a reduced number of key product factors. Their immediate questions concerned what factors are "competing" between each other and to what extent. Moreover, their intuition and knowledge of the market supports the idea that preferences for key factors, like brands, are evolving in time. However, they usually measure these time variations based on market wide "top of mind" surveys while the use of sales data for this type of analysis is rare. Hence, the modeling of the evolution in brands-preferences based in market shares data, they argue, is one of the key and most valuable aspects of our approach.

4.7 Tables and Figures

	Posterior	HPDR	
	Mean	1%	99%
$\tau^2_{m=1}$	0.636	0.618	0.698
$\tau^2_{m=2}$	0.640	0.624	0.670
$\tau^2_{m=3}$	0.595	0.580	0.626
$\tau^2_{m=4}$	0.659	0.641	0.717
$\tau^2_{m=5}$	0.618	0.598	0.657
$\tau^2_{m=6}$	0.580	0.565	0.598
$\tau^2_{m=7}$	0.657	0.640	0.711
$\tau^2_{m=8}$	0.662	0.647	0.755
$\tau^2_{m=9}$	0.641	0.624	0.682
$\tau^2_{m=10}$	0.630	0.615	0.680

Notes: The true value of τ^2_m is equal to 0.64 for all m. HPDR stands for Highest Posterior Density Region.

Table 4.1: Simulation Experiment: Posterior Distribution of the Variance of the Demand Shocks

	Posterior Mean	HPDR 1%	HPDR 99%
$\sigma^2_{m=1}$	0.107	0.082	0.148
$\sigma^2_{m=2}$	0.120	0.096	0.157
$\sigma^2_{m=3}$	0.118	0.088	0.144
$\sigma^2_{m=4}$	0.120	0.094	0.155
$\sigma^2_{m=5}$	0.136	0.104	0.182
$\sigma^2_{m=6}$	0.134	0.102	0.170
$\sigma^2_{m=7}$	0.128	0.096	0.170
$\sigma^2_{m=8}$	0.116	0.090	0.153
$\sigma^2_{m=9}$	0.121	0.090	0.160
$\sigma^2_{m=10}$	0.128	0.098	0.170

Notes: The true value of σ^2_m is equal to 0.16 for all m. HPDR stands for Highest Posterior Density Region.

Table 4.2: Simulation Experiment: Posterior Distribution of σ^2_m

	Posterior	HPDR		Real
	Mean	1%	99%	Value
Brand A	1.950*	1.081	4.089	2.0
Brand B	0.435*	0.209	3.446	2.0
Brand C	3.036*	1.958	3.755	2.0
Brand D	1.127*	0.890	3.514	2.0
Brand E	7.890*	5.126	8.983	8.0
Brand F	3.929	3.119	4.702	8.0
Brand G	3.066	2.421	4.500	8.0
Brand H	1.992	1.776	3.789	8.0
Brand I	2.295	1.817	3.617	4.0
Brand J	2.219	1.938	3.603	4.0
Attribute b	4.827	4.270	5.316	4.0
Attribute c	1.990	1.463	3.636	4.0
Attribute d	1.473*	0.732	3.202	2.0
Attribute e	2.988	2.693	3.490	2.0
Price	0.866	0.543	1.510	2.0
Promotion	1.533*	1.112	2.550	2.0

Note: * means that the real value is included in the HPDR. HPDR stands for Highest Posterior Density Region.

Table 4.3: Simulation Experiment: Posterior Distribution of the elements of D^2.

	Posterior	HPDR	
	Mean	1%	99%
$\tau^2_{m=1}$	5.329	0.606	53.233
$\tau^2_{m=2}$	7.518	4.782	11.322
$\tau^2_{m=3}$	0.814	0.659	1.034
$\tau^2_{m=4}$	0.895	0.684	1.172
$\tau^2_{m=5}$	1.500	1.127	2.042
$\tau^2_{m=6}$	1.284	0.989	1.715
$\tau^2_{m=7}$	0.831	0.577	1.281
$\tau^2_{m=8}$	0.638	0.574	0.734
$\tau^2_{m=9}$	0.838	0.754	1.013
$\tau^2_{m=10}$	0.467	0.395	0.589
$\tau^2_{m=11}$	1.015	0.805	1.322
$\tau^2_{m=12}$	5.664	1.632	65.241
$\tau^2_{m=13}$	2.631	1.906	3.652
$\tau^2_{m=14}$	0.752	0.672	0.901
$\tau^2_{m=15}$	1.917	1.490	2.572
$\tau^2_{m=16}$	1.439	1.316	1.607
$\tau^2_{m=17}$	4.144	0.602	44.067
$\tau^2_{m=18}$	1.612	1.328	2.101

Note: HPDR stands for Highest Posterior Density Region.

Table 4.4: Application: Posterior Mean and HPDR of the τ^2_m.

	Posterior	HPDR	
	Mean	1%	99%
$\sigma^2_{m=1}$	0.0149	0.0090	0.0249
$\sigma^2_{m=2}$	0.0200	0.0102	0.0397
$\sigma^2_{m=3}$	0.0628	0.0357	0.1213
$\sigma^2_{m=4}$	0.0384	0.0206	0.0650
$\sigma^2_{m=5}$	0.0128	0.0080	0.0276
$\sigma^2_{m=6}$	0.0180	0.0101	0.0293
$\sigma^2_{m=7}$	0.0361	0.0208	0.0600
$\sigma^2_{m=8}$	0.0322	0.0190	0.0563
$\sigma^2_{m=9}$	0.0390	0.0202	0.0654
$\sigma^2_{m=10}$	0.0339	0.0196	0.0611
$\sigma^2_{m=11}$	0.0557	0.0272	0.1021
$\sigma^2_{m=12}$	0.0131	0.0072	0.0236
$\sigma^2_{m=13}$	0.0204	0.0113	0.0425
$\sigma^2_{m=14}$	0.0252	0.0150	0.0450
$\sigma^2_{m=15}$	0.0132	0.0088	0.0230
$\sigma^2_{m=16}$	0.0255	0.0139	0.0429
$\sigma^2_{m=17}$	0.0123	0.0073	0.0249
$\sigma^2_{m=18}$	0.0147	0.0087	0.0202

Note: HPDR stands for Highest Posterior Density Region.

Table 4.5: Application: Posterior Mean and HPDR of the σ^2_m.

		Posterior	HPDR	
		Mean	1%	99%
Market 1	Size A	0.811	-0.098	1.766
	Size B	-1.835	-3.025	-0.577
	Size C	0.268	-0.643	1.164
	Size D	-0.513	-2.131	1.174
	Size E	-2.042	-3.053	-1.017
	Price	-2.684	-3.925	-1.529
	Promotion	-2.380	-6.455	1.856
Market 2	Size A	0.120	-0.207	0.438
	Size B	-0.934	-1.204	-0.669
	Size D	0.414	0.059	0.739
	Size E	-0.218	-0.660	0.191
	Price	-0.904	-1.472	-0.309
	Promotion	-2.714	-4.229	-0.894
Market 3	Size A	0.749	0.397	1.093
	Size B	0.237	-0.087	0.430
	Size C	-0.594	-1.015	-0.224
	Size D	-0.641	-0.877	-0.367
	Size E	-0.545	-0.821	-0.271
	Price	-0.593	-0.924	-0.211
	Promotion	-3.388	-4.764	-2.079

Note: HPDR stands for Highest Posterior Density Region.

Table 4.6: Application: Posterior Mean and HPDR of the Fixed Elements of f_m (size and price and promotion coefficients) for 3 out 18 markets

	Posterior	HPDR	
	Mean	1%	99%
D_1^2	1.436	0.801	2.493
D_2^2	0.798	0.653	0.896
D_3^2	1.341	1.184	1.619
D_4^2	0.831	0.628	1.461
D_5^2	0.293	0.249	0.335
D_6^2	0.874	0.739	1.022
D_7^2	1.853	0.970	4.412
D_8^2	0.500	0.443	0.575
D_9^2	4.648	1.509	9.801
D_{10}^2	0.395	0.327	0.457
D_{11}^2	1.197	0.787	1.718
D_{12}^2	0.569	0.466	0.727
D_{13}^2	0.556	0.478	0.603
D_{14}^2	0.628	0.522	0.700
D_{15}^2	10.625	3.911	21.916
D_{16}^2	0.432	0.329	0.502
D_{17}^2	0.195	0.160	0.238
D_{18}^2	0.990	0.750	1.257
D_{19}^2	3.249	1.824	5.073
D_{20}^2	0.221	0.182	0.276
D_{21}^2	0.337	0.248	0.378
D_{22}^2	0.602	0.540	0.698
D_{23}^2	7.294	5.028	12.136
D_{24}^2	0.700	0.623	0.794
D_{25}^2	2.361	1.697	2.710
D_{26}^2	0.754	0.654	0.955
D_{27}^2	0.643	0.522	0.732
D_{28}^2	0.558	0.472	0.667
D_{29}^2	0.579	0.468	0.672
D_{30}^2	0.587	0.474	0.797
D_{31}^2	0.590	0.443	0.777
D_{32}^2	0.624	0.490	0.811

Note: HPDR stands for Highest Posterior Density Region.

Table 4.7: Application: Posterior Distribution of the Elements of the D^2 matrix, where $\Psi = DSD$.

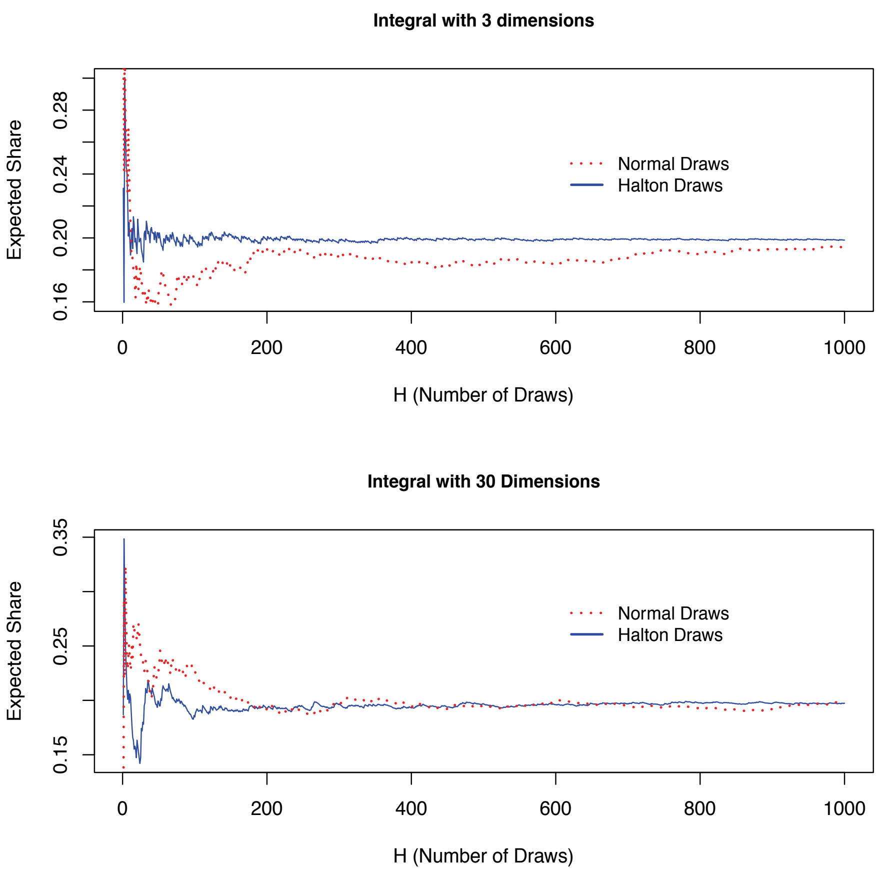

Figure 4.1: Performance of Halton Based Normal Draws versus Normal Draws

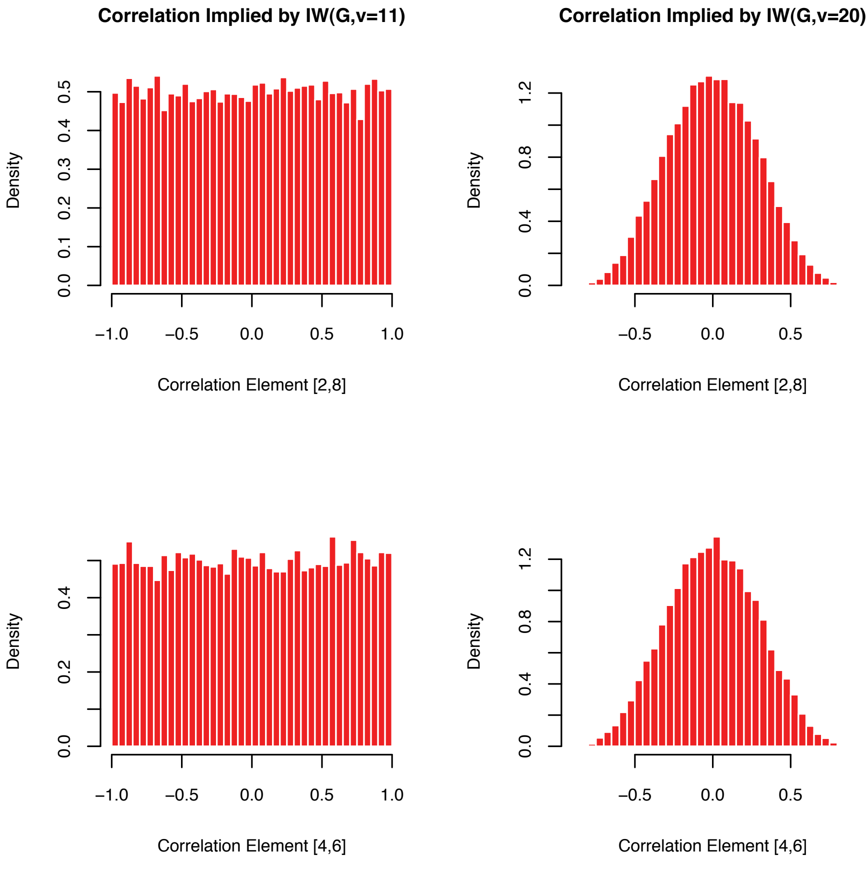

Figure 4.2: Prior Correlations for Different Elements of Ψ. The degrees of freedom for the Wishart Distribution v are set to 11 for the left panel and 20 for the right panel.

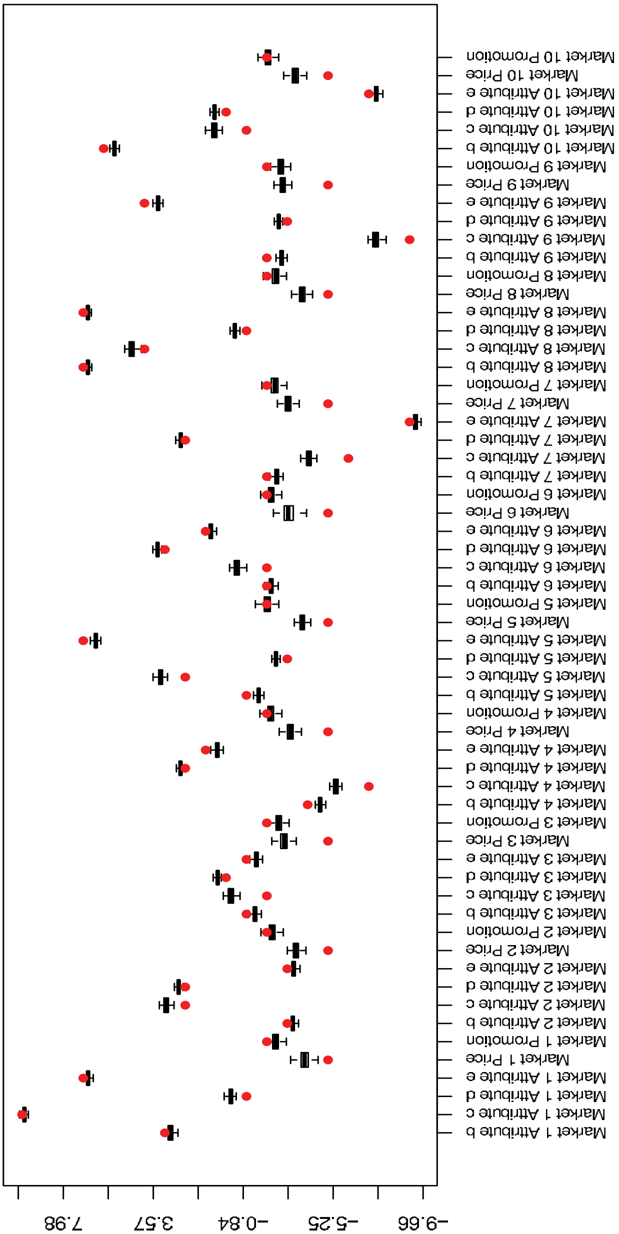

Figure 4.3: Simulation Experiment: Real (Circles) versus the Posterior Distribution (Box-plots) of the Fixed Coefficients

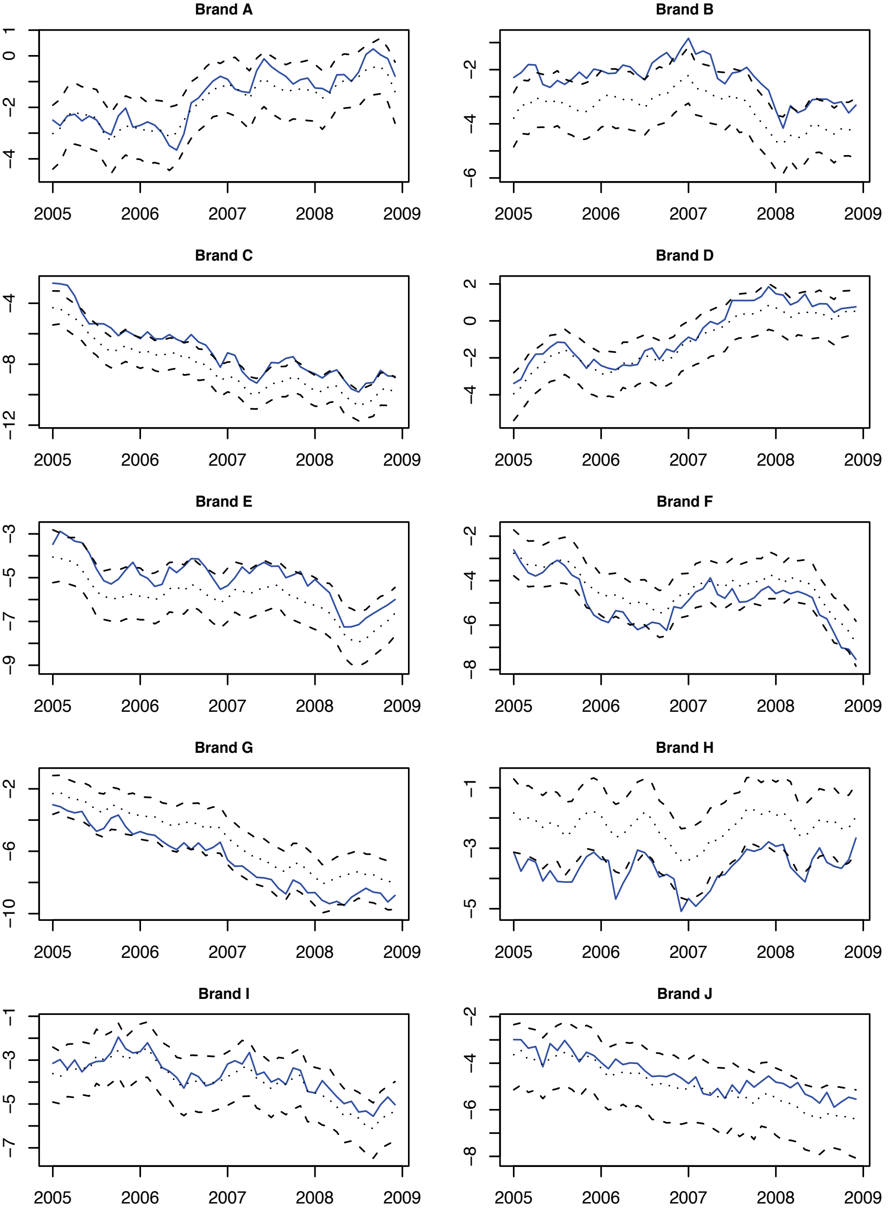

Figure 4.4: Simulation Experiment: Real (Solid Line) versus Posterior Mean (Dots) and the 99% HPDR (Dashed Lines) of the Time-Varying Brand Coefficients at Market 5

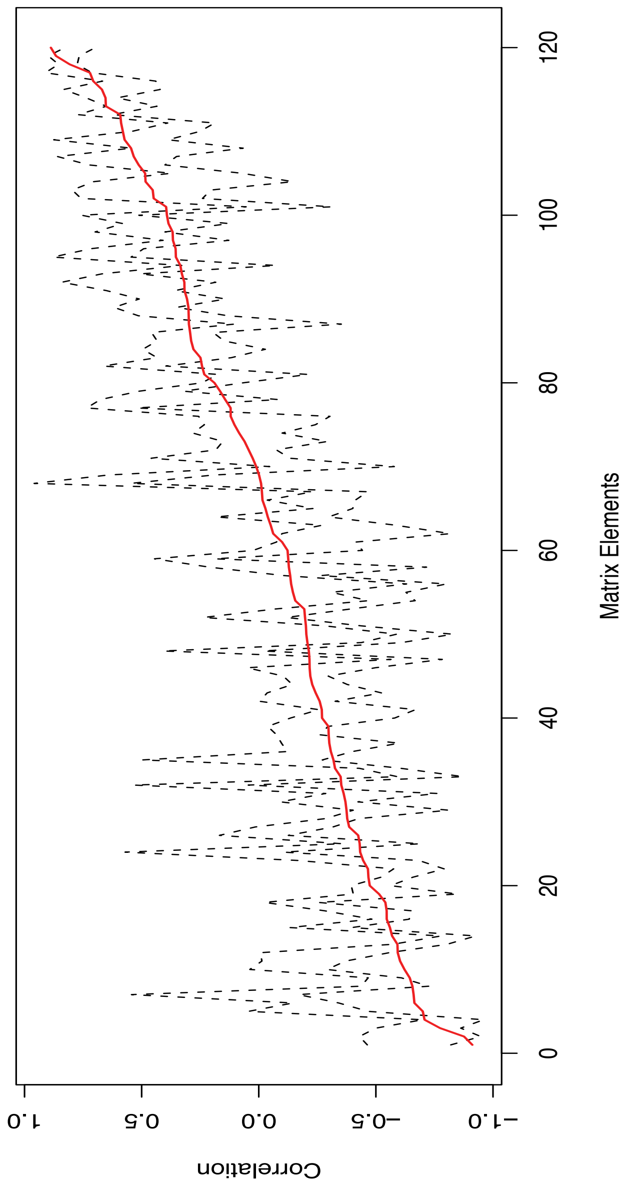

Figure 4.5: Simulation Experiment: Real (Solid Line) versus Posterior 99% HPDR (Dashed Lines) of All Elements in the Correlation matrix *R*.

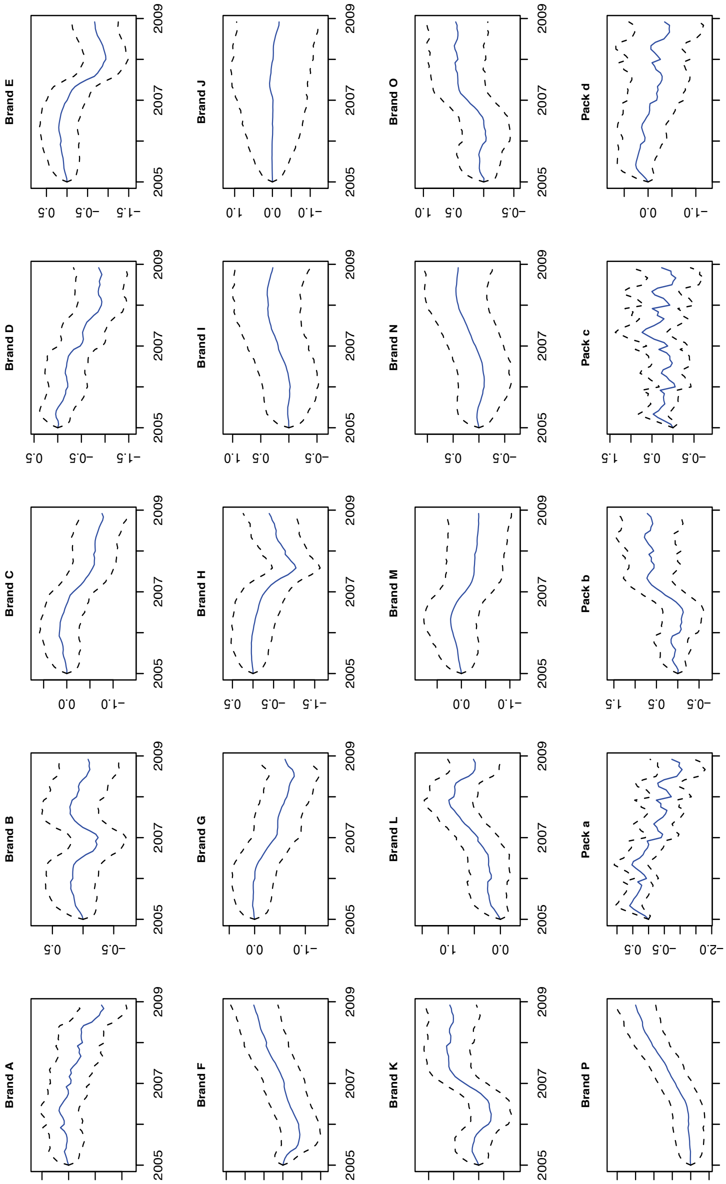

Figure 4.6: Application: Time-Profile Relative to First Period of 8th Market Time-Varying Factors f^m (Solid Lines) and their 99% HPDR (Dashed Lines).

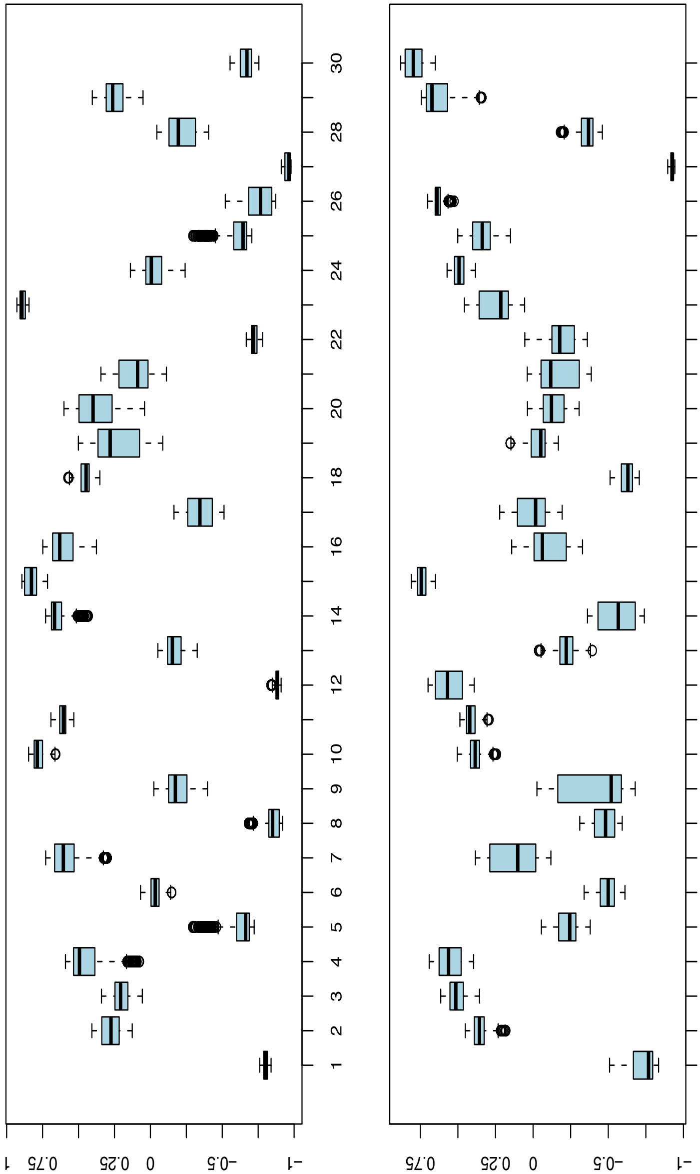

Figure 4.7: Application: Distribution of 60 Correlation Elements of the Ψ matrix size 32×32

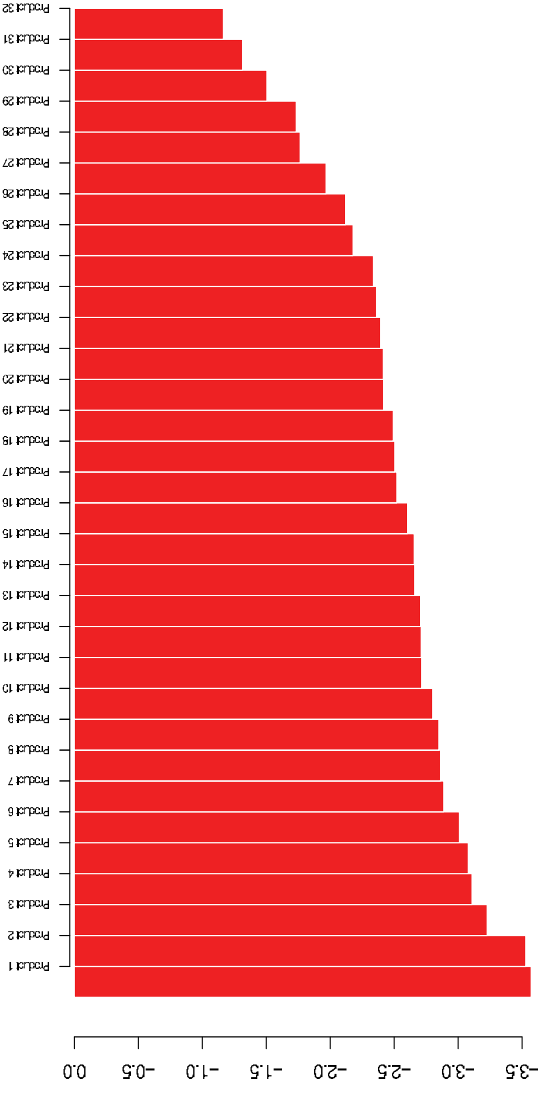

Figure 4.8: Own Price Elasticity for Products at Market 2

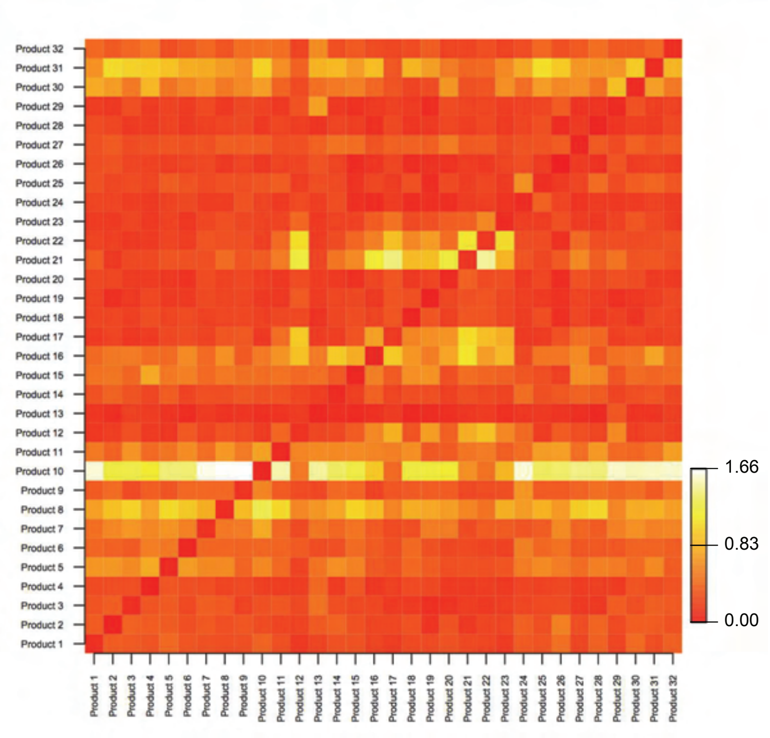

Figure 4.9: Cross-Price Elasticities at Market 2

4.A Appendix

Iterative BLP Procedure

We use the iterative procedure proposed in BLP to obtain the $\mu_t = (\mu_{1t}, \ldots, \mu_{Jt})$. Note that, for convenience, we omit the market indicators m here. The procedure consists of the following steps. First we obtain H draws of υ_i. To this end we write the d-th draw as $\upsilon_i^d = \Psi^{1/2}\zeta^d$ where ζ^d is draw from a joint normal distribution which we obtain using scrambled Halton draws. Given these draws and some initial value for μ_t we can compute the implied market shares $\hat{s}_t$. Given the shares, both real s_t and implied $\hat{s}_t$, we can use the contraction mapping

$$\mu_t^{new} = \mu_t^{old} + \log(s_t) - \log(\hat{s}_t) \tag{4.26}$$

to obtain a new value for μ_t. We repeat the contraction mapping computing the implied shares $\hat{s}_{jt}$ as

$$\left(\sum_{i=1}^{H} \frac{\exp(\mu_{jt}^{old} + \lambda_{jt}'\upsilon_i)}{\exp(\mu_{0t}) + \sum_k \exp(\mu_{kt}^{old} + \lambda_{kt}'\upsilon_i)} \right) / H \tag{4.27}$$

for $j = 1, \ldots, J$ and $t = 1, \ldots, T$ and we stop the contraction mapping when the values of μ_t^{new} and μ_t^{old} converge.

Note that we include and solve for the outside good utility μ_{0t} in the contraction mapping iterations. We discovered that the precision of the contraction mapping is higher when we iterate over the utilities of all products together with the utility of the outside good.

Computing Elasticities

We use the following definition to compute the price elasticities φ_{jl}^m between product j and l in market m conditional on all model parameters:

$$\varphi_{jl}^m = \frac{p_{lt}^m}{\mathrm{E}[s_{jt}^m]} \frac{\partial \mathrm{E}[s_{jt}^m]}{\partial p_{lt}^m} = \frac{p_{lt}^m}{\int s_{jt}^m \pi(\eta_t; 0, \tau_m^2 \mathbf{I}) d\eta_t^m} \times \int \frac{\partial}{\partial p_{lt}^m} s_{jt}^m \pi(\eta_t | \tau_m^2) d\eta_t^m \tag{4.28}$$

where s_{jt}^m is defined in (4.7) and

$$\frac{\partial}{\partial p_{lt}^m} s_{jt}^m = \begin{cases} -\beta^m \int s_{ijt}^m s_{ikt}^m \phi(\upsilon_i^m; 0, A^m \Psi A_m') d\upsilon_i^m & \text{if } k \neq j \\ \beta^m \int s_{ijt}^m (1 - s_{ikt}^m) \phi(\upsilon_i^m; 0, A^m \Psi A_m') d\upsilon_i^m & \text{if } k = j, \end{cases} \tag{4.29}$$

and β^m is the price coefficient in market m. Finally, to obtain the posterior distribution of the price elasticities we average (4.28) over the posterior draws for all parameters.

Chapter 5

Finding the Influentials that Drive the Diffusion of New Technologies

In this chapter we consider the diffusion of similar technologies in a single market composed of many locations. We address the identification of the influential locations that drive the aggregate sales of these new technologies based on aggregate sales data and location specific online search data.

In this chapter we put forward a model where aggregate sales are a function of the online search of potential consumers at many locations. We argue that a location may be influential because of its power to drive aggregate sales and this power may be dynamic and evolving in time. Second, the influential locations may produce spillover effects over their neighbors and hence we may observe clusters of influence. We apply Bayesian Variable Selection (BVS) techniques and we use Multivariate Conditional Autoregressive Models (MCAR) to identify influentials locations and their clustering.

We apply our methodology to the video-game consoles market and to new search data of Google Insight. More precisely, we study the influential locations that drive the sales growth of the Nintendo Wii, the Sony PS3 and Microsoft Xbox 360. Specifically, we study the diffusion of these technologies at four different stages of their life-cycle. In this way, we can identify the group of influential locations and its composition in different sub-periods.

Our results indicate that the influential locations and their economic value (measured by search elasticities) vary over time. Moreover, we find significant geographical clusters of influential locations and the clusters composition varies during the life-cycle of the consoles. Finally, we find weak evidence that demographics explain the probability of a location to be influential. The main managerial implication of our results is the notion that the group of influential locations and their clustering varies during the life-cycle of a technology. Hence, managers should aim to identify the identity plus the locations and the dynamics of influentials.

5.1 Introduction

An important topic related to the diffusion of new technologies is the identification of influentials. Influentials play an important role as opinion leaders and trend setters and they critically affect the speed of adoption of new technologies (van den Bulte and Joshi, 2007).

Recent attention is being given to the identification of the location and identities of these influentials. In the literature, influentials are defined as individuals or groups of individuals that influence the behavior of others in a significant way. Their influence has been studied at the individual level (Trusov et al., 2010), at the firm level (Albuquerque et al., 2007) and at the country level (van Everdingen et al., 2009). Influentials may have a specific location in a social network (Trusov et al., 2010; Christakis and Fowler, 2009; Cho and Fowler, 2007) or a specific physical location (Choi et al., 2009; Goldenberg et al., 2009). Their influence can be limited to a few others (Christakis and Fowler, 2009, page 28) but their impact may also exceed national boundaries (van Everdingen et al., 2009).

In this article we study the diffusion of a number of similar and competing technologies and we address the identification of the influential locations that drive the aggregate sales of these new technologies. We put forward a model where sales are a function of the online search registered at many different locations. We will refer to this model as the sales-search model. We know that consumers search for technologies (or products) online and we posit that online search should be a good predictor of sales. However, people in many different locations search for products while only the consumers living in a subset of these locations may be the key groups driving the sales of new technologies. Moreover, the influential locations may not always be the same. And, the cross-influence among locations may be important and time-varying or fixed in time.

We present an approach that is new to the marketing literature and we study new search data obtained from Google Insight. Our novelty is that we use the sales-search model together with Bayesian Variable Selection techniques to select the locations that are most likely driving the aggregate sales of these three new technologies. We use this methodology because there are many possible important locations and a straightforward choice between them is not possible. In addition, we present a second model with Multivariate Conditional Autoregressive priors (known as MCAR priors) to study the cross-location influence, the significance of spatial clustering of influential locations and the competing relationships between technologies. We will refer to this model as the spatial model.

Our data consists of the aggregate weekly sales of the Nintendo Wii, the PlayStation 3 and the Microsoft Xbox 360 for the entire US market and online search data for each of these products. The online search data were obtained from Google Insight and these data consist of weekly indicators of online search for each of these technologies in each US state. The data cover a period from the launch time of each technology up to February 2010 (approximately four years) for both the sales and the online search data. This dataset is attractive because it allows us study three very successful technologies that receive worldwide interest. These three products were marketed simultaneously in all US states and this fact allows us to discard the explanation that a region may become influential because its products were available at an earlier introduction date relative to other regions.[1] Moreover, these technologies have unique names and they have kept these unique names for long periods of time and therefore we can obtain reliable online search data for all US states.[2] The sales data we observe can be easily classified in different periods of the products' life-cycle and we will identify the influential locations at these product life-cycle stages. We base these life-cycle stages on Rogers (2003) who suggests that innovations are characterized by five periods when different groups of people (innovators, early adopters, early majority, late majority and laggards) adopt an innovation. In this way we will be able to uncover the location of influential groups of adopters at different life-cycle phases of the products. Our results suggest that the influential regions driving aggregate sales differ across the life-cycle of a technology. Moreover, our approach uncovers geographical clustering of both influential and not influential regions. Influential regions seem to be close to each other but we find that their influence and the geographical clustering varies over time. In addition, we find only a weak association between demographic information and the probability that a region is influential. Finally, our results indicate that a 10% increase in local online search translates

[1]For example, the launch time of the products studied by van Everdingen et al. (2009) differs across countries.

[2]Note that it is impossible to obtain state level sales data. We made inquiries at different market research firms, including NPD group, and to our knowledge there are no firms collecting these data.

on average into a 1.5% percent increase in global sales but this number varies across regions and diffusion periods and its range goes from 0 up to 3%.

The plan of the chapter is as follows. In Section 5.2 we discuss previous literature and its relationship to our work. In Section 5.3 we present our methodology. Later in Section 5.4 we give details about our data and some specific details regarding our model. In Section 5.5 we present our results and finally in Section 5.6 we conclude the chapter. The statistical methodology that we use is presented in detail in Section 5.A and Section 5.A.

5.2 Literature Review

The literature related to our work can be classified into micro-studies of adoption, like Choi et al. (2009), Goldenberg et al. (2009), Trusov et al. (2010), Garber et al. (2004) and Jank and Kannan (2005), and into macro-studies of technology diffusion, like van Everdingen et al. (2009), Albuquerque et al. (2007) and Putsis Jr et al. (1997).

van Everdingen et al. (2009) examine the global spillover effects of product introductions and take-offs. They find that the product take-off in a country can help to predict the take-off of the same product in different countries. In addition, they report asymmetric patterns of influence and foreign susceptibility. The heterogeneity in the spill-over effects is significantly explained by economic and demographic characteristics. Moreover, van Everdingen et al. (2009) discuss briefly the time dimension of influence. Their results suggest that there are countries that have a large impact on others late in the diffusion process, while other countries may have a smaller influence but sooner. Albuquerque et al. (2007) study the global adoption of two ISO certification standards and their results indicate that cross-country influence is important and it improves the fit of their model. They find that the role of culture, geography and trade in the adoption process is different across the ISO standards. They use a multi-country diffusion model and therefore they assume that a firm's adoption is influenced by previous cumulative number of adoptions by other firms in different countries. Therefore, the global cumulative adoptions of ISO standards foster more adoptions. Albuquerque et al. (2007) also find that the influence of cumulative past adoptions is stronger among firms close to each other or between firms in neighboring countries. Finally, Putsis Jr et al. (1997) study cross-country and inter-country diffusion patterns and they report important cross-country influence on diffusion. Their findings suggest that each country's influence varies from product to product.

The micro diffusion studies have documented the role and economic value of influential people in a social network (Trusov et al., 2010; Goldenberg et al., 2009) and the formation of spatial clusters (Garber et al., 2004; Choi et al., 2009; Jank and Kannan, 2005). The study of Garber et al. (2004) deals with the spatial distribution of adoption. They discovered that the spatial pattern at early stages of the diffusion of a technology is an accurate predictor of new product success. They argue that spatial clustering is a sign of imitation and therefore if the spatial distribution of adoption shows clusters it is very likely that the diffusion process will continue and sales will eventually take off. They compare the spatial distribution of adoption against a uniform distribution of adoption and they find that successful products show an early spike of divergence between these two distributions (cross-entropy) while the cross-entropy of product failures remains relatively constant and low.

More recently, Choi et al. (2009) studied the temporal and spatial patterns of adoption in Pennsylvania and they discovered that the spatial clusters of adoption change over time and that the cross-region (cross zip code) influence decays over time. In the same way, Jank and Kannan (2005) report spatial clusters of customers with the same price sensitivity and preferences and they use spatial random effects to capture the geographical variation in preferences. The study of Hofstede et al. (2002) is focused in identifying spatial country and cross-country segments and they find evidence of contiguous and spatial clustering of consumer preferences. They argue that the spatial dependence in preferences should be useful to define distribution and marketing decisions across countries. Bradlow et al. (2005) provide an overview of spatial models and their relationship to marketing models. Finally, Trusov et al. (2010) and Goldenberg et al. (2009) suggest that influentials can have a significant economic value and they may foster the diffusion of new technologies.

In this chapter we explore the time dimension and the spatial structure of influence at the level between micro and macro, that is at the regional level within a country. The objective of van Everdingen et al. (2009) and Albuquerque et al. (2007) is to identify the cross-country influence while our objective is to discover whether a region is influential and when it is influential. In contrast with previous research, in our study a region may be influential initially while later it may exert no influence at all or the other way around. That is, we consider the influence across the life-cycle of the products' diffusion while previous research has not focused particularly on this aspect. Moreover, the Bayesian Variable Selection technique that we use to detect influentials also distinguishes our study from previous work at a technical level. Finally, the visual inspection of our results suggests important geographical clusters of influential regions and we study whether these geographical clusters of influence are statistically relevant. For this latter purpose, we fit a spatial model with MCAR priors and perform tests to detect spatial clusters. It is the univariate version of this prior that has recently been applied in some marketing studies, an example is Duan and Mela (2009). The MCAR prior can incorporate both the spatial structure of the data as well as the relationship between technologies. To our knowledge, we are the first to use an MCAR prior on a marketing application while it must be mentioned that this prior is frequently used in bio-statistics and environmental studies.

5.3 Methodology

The approach we use consists of two main parts. First, in Section 5.3.1 we describe how we use Bayesian Variable Selection techniques to identify the regions and the sub-periods during which each region is likely to drive aggregate sales. The Bayesian Variable Selection technique will let us compute the posterior probability that a region is influential for any given sub-period. In Section 5.3.2 we specify a second model to study these posterior probabilities and our main objective in this section is to test whether there are important spatial clusters or demographic variables explaining these inclusion probabilities.

5.3.1 The Sales-Search Model

We observe the aggregate sales y_{it} of $i = 1, \ldots, M$ technologies at time $t = 1, \ldots, T$. We also observe the online search s_{ijt} for each of these i technologies at J different locations for $j = 1, \ldots, J$ and time periods $t = 1, \ldots, T$. In addition, s_{ijtn} will refer to the search observed at location j at a time t that is included in sub-period n, for $n = 1, \ldots, N$. We define sub-periods of diffusion because we are interested in studying the early, mid and late diffusion of the technologies.

The sales equation is specified as

$$y_{it} = \sum_j \sum_n \beta_{ijn} s_{ijtn} + \epsilon_{it} \text{ where } \epsilon_{it} \sim N(0, \sigma_i^2). \tag{5.1}$$

where both y_{it} and s_{ijtn} are in logs; sales are measured in hundred thousands and search is measured as an "interest indicator" and its range goes from 10 to 110. We give more details about the data in Section 5.4. We specify a technology i, sub-period n (for $n = 1, \ldots, N$) and region j specific coefficient β_{ijn} and the error term ϵ_{jt} is assumed to be normal with zero mean and variance σ_i^2.

This specification sums over all sub-periods n and locations j but estimating such a model may be impossible when the total number of regressors $J \times N$ is large relative to T. Note that in practice $J \times N$ can be even much larger than T. Moreover, it is very likely that many of the $\beta_{ijn} = 0$ because of the likely correlation among the s_{ijn} and the fact that some locations may simply do not drive sales. Hence, we need to select a subset location specific regressors that consists of the best set of all possible regressors. We will call the set of all possible regressors X and we will use X_γ to refer to the subset of best regressors. We will call q_γ to the total number of elements in X_γ and p to the total number of elements in X. That is, $X_\gamma \subset X$ and X is a set containing s_{ijn} for $j = 1, \ldots, J$ and $n = 1, \ldots, N$. The purpose is to select a model that sums only over this subset. Therefore we specify

$$y_{it} = \sum_j \sum_n \gamma_{ijn} \beta_{ijn} s_{ijn} + \epsilon_{it} \text{ where } \epsilon_{it} \sim N(0, \sigma^2) \tag{5.2}$$

as the sales equation where γ_{ijn} is a technology and region sub-period specific indicator that takes the value of 1 if s_{ijn} is in the subset X_γ and zero otherwise. Note that JN potential regressors result in 2^{JN} possible subsets and vectors γ_i where $\gamma_i = (\gamma_{i11}, \ldots, \gamma_{iJN})'$.

One could suggest for equation (5.2) that we could also sum over i on the right hand side and not only j and n. That is, the sales of a technology could be a function of the search for all technologies in the market. However, in our application there are over 2.57×10^{61} (that is $2^{51\times4}$ where 51 is the number of locations and 4 is the number of sub-periods) possible subsets of regressors and if we were to sum over i there would be more than 1.69×10^{184} (that is $2^{3\times52\times4}$) subsets of models. That is 6.61×10^{122} more subsets. Therefore, we study the relationship between technologies with a different model and we discuss this second model later in this section. A second issue is that sales are a function of search while at the same time search may be a function of sales. We are aware of this possible endogeneity of sales and search but as we are using local indicators for search and aggregate measures for sales we believe the endogeneity between them should be relatively weak. Finally, the right hand side could contain lags of the search indicators. However, the inclusion of lags forbids us to compare the inclusion reason across locations. For example, a location may be selected because it has an important lagged effect while another location because of its contemporaneous effect on sales. We restrict the model to a contemporaneous relationship between sales and search to be able to use the probability of a location regressor to be in X_γ at a later stage in the spatial model.

We use Bayesian Variable Selection (BVS) as presented in George and McCulloch (1997) and Chipman et al. (2001) to select the best subset of regressors. To use BVS we need proper priors, we specify $\pi(\beta_i|\sigma_i, \gamma_i)$ as in Equation (5.5) and $\pi(\sigma_i^2|\gamma_i)$ as in equation (5.7); these are the prior distributions of β_i coefficients and the variance σ_i^2 where $\beta_i = (\beta_{i11}, \ldots, \beta_{iJN})$ and we specify the prior distribution of the indicators $\pi(\gamma_i)$. We use equations (5.9) and (5.10) to define the prior on γ. BVS is an attractive technique because we can draw inferences on the probability of inclusion for each potential regressor in model (5.2). That is, we can draw inferences on the posterior distribution of the indicators given the data $\pi(\gamma_i|y_i)$ where $y_i = (y_{it}, \ldots, y_{iT})'$. We estimate model (5.2) for each of the technologies separately and details of our estimation approach are presented in the Appendix. In the Appendix we drop the sub-index i because we use the same prior specification for all technologies.

5.3.2 The Spatial Model

The indicator vector γ_i is composed of location and sub-period indicators and based on BVS we can compute for each element of the vector γ_i the probability that it equals one. That is, we can compute each region's posterior probability to be included at any sub-period and this posterior is available for each of the technologies. We will refer to the logit transformation[3] of this posterior probability as $\bar{p}_{ijn}$ where as before i refers to the technology, j to the location and n is the sub-period index.

Our objective is to test whether the variation in inclusion probabilities is explained by demographic variables and whether there are significant spatial effects in these inclusion probabilities. Hence, we propose a model where the posterior probabilities of inclusion depend on a set of covariates Z_n and their corresponding coefficients δ_n plus spatial effects Φ_n and some noise ε_n. We propose that

$$\bar{P}_n = Z_n\theta_n + \Phi_n + \varepsilon_n \tag{5.3}$$

where $\bar{P}_n = (\bar{p}'_{1n}, \ldots, \bar{p}'_{Mn})$, $\bar{p}'_{in} = (\bar{p}_{i1n}, \ldots, \bar{p}_{iJn})$. That is, $\bar{P}_n$ is a $J \times M$ matrix with the inclusion probabilities of each of the J locations for each technology in M columns. Z_n are covariates available for period n where Z_n is a $J \times K$ matrix where K is the number of covariates. We assume $\theta_n = \iota \otimes \delta_n$ is a $K \times M$ matrix with coefficients where ι is a row vector of ones of size M and δ_n is a $K \times 1$ vector of coefficients. $\Phi_n = (\phi'_{1n}, \ldots, \phi'_{Mn})$, $\phi'_{in} = (\phi_{i1n}, \ldots, \phi_{iJn})$ and $\varepsilon_n = (\varepsilon'_{1n}, \ldots, \varepsilon'_{Mn})$ with $\varepsilon'_{in} = (\varepsilon_{i1n}, \ldots, \varepsilon_{iJn})$. Both Φ_n and ε_n are $J \times M$ matrices.

The spatial effects Φ are a function of the relationships between technologies and the neighborhood structure of the market. The Φ matrix is composed of one spatial effect for each location and technology. Each spatial effect, in general terms, depends on the spatial effects of all technologies at neighboring locations. Hence, the spatial effects reflect spatial clustering but they do not detect the direction of influence between locations. This property of the spatial effects is specified in a prior distribution that depends on Λ, Ψ and ρ where Λ is a $M \times M$ matrix with the covariance structure between the technologies, Ψ is a $J \times J$ matrix that measures the neighborhood or the spatial structure of the market and ρ is a parameter that measures spatial auto-correlation. The element Ψ_{kl} is either a fixed distance between location k and l or an indicator that takes a value of 1 if the location k is a neighbor of l and zero otherwise. In the Appendix we provide details on how we draw inference about ρ, Λ, δ_n and the covariance matrix

[3]The function is log(p/(1-p)). A second transformation may be log(-log(p)). We tested both transformations and our results are similar.

associated with ε_n. Note that Ψ is a fixed matrix with the neighborhood structure and hence we do not estimate it. We give more details about Ψ in the next section.

Next, we use this specification to explore if there are significant spatial effects Φ in the posterior probabilities of inclusion for each region during each sub-period n and if there is a relationship between the inclusion probabilities between technologies after controlling for the covariates in Z_n. Note that in the equation (5.3) we are pooling all technologies $i = 1, \ldots, M$ together. The reason we pool technologies together is that their inclusion probabilities may be related to each other. For example, Texas could be the driver of growth for one technology but not for all technologies. That is, technologies may be competing against each other when the sign of the covariance terms in the Λ matrix are negative.

5.4 Data and Modeling Details

Weekly search indicators are available online from Google Insight for all US states and the weekly series of sales data for the video-game consoles were obtained from VGchartz.com. The data of VGchartz follows very closely the monthly figures of the NPD group. We use the latest (year 2000) demographic information of the US Census Bureau for all US states.

In Figure 5.1 we present a printed screen with the exact keywords that we used to retrieve the search data from Google Insights for Search (`http://www.google.com/insights/search/`). In Table 5.1 we provide the R code to automatically retrieve the data from `http://www.vgchartz.com/`.

To estimate the parameters of equation (5.2) we used MCMC and the chain ran for 210 thousand iterations and we discarded the first 10 thousand. The equation that we used includes a spline term that captures the seasonal fluctuation of y_i and its overall level. We fit a smoothing spline of y_i as a function of time and we use 10 degrees of freedom as the smoothing parameter; we refer to Hastie et al. (2001, page 127-137) for mode details on fitting smoothing splines. Sloot et al. (2006) also use spline terms to capture seasonal fluctuations. The spline term is always included on the right-hand side of the model and we do not use BVS on this term. Finally, note that we used the logs of y_i and the s_{ijn} and that y_{it} are the sales of the technology

i at the end of week t and s_{ijt} is the online search index for the technology i at state j during the week t.

Next, we use MCMC to estimate the parameters of equation (5.3) and the chain ran for 2000 iterations and we discarded the first 1000. We used much less draws than before because convergence for a linear model is quite fast. We run the estimation for each sub-period separately and therefore we estimated the parameters of equation (5.3) for each period.

We divide the sales data of each consoles in four periods of equal length. These periods roughly correspond with the first four stages of adoption proposed by Rogers (2003). It is likely that in practice the length of each period varies per product or industries. For example, we know that the time to take-off is different across countries while within a country the take offs tend to occur at a systematic time difference relative to other countries (van Everdingen et al., 2009; Golder and Tellis, 1997; Tellis et al., 2003). Additionally, we choose periods of equal length to be able to compare the influential locations across products for exactly the same period of time. In this way we can naturally make cross-product comparisons.

We estimate equation (5.2) and equation (5.3) separately because we prefer not to impose any spatial structure on the prior probability of including regressors in the prior for the indicator variables, that is $\pi(\gamma)$. We estimate equation (5.3) for each life-cycle stage. The disadvantage of treating equations (5.2) and (5.3) separately is that the uncertainty of the first model is not taken into account in the second model. A technical reason to keep the estimation of these equations separately is that the posterior probabilities of inclusion are computed using the full MCMC chain and therefore we know them only at the end of the estimation. However, the most important reason to keep the estimation in two steps is not to impose a priori a spatial structure in the inclusion probabilities. In this way, we leave the task of testing for spatial clustering as a second step and we may be able to provide stronger evidence of any spatial structure.

We checked for convergence of the MCMC chains visually. We give more details about the estimation approach and about the MCAR models in the Appendix.

5.5 Results

In this section we first discuss the results for the sales-search model in equation (5.2) and then for the spatial model in equation (5.3).

5.5.1 Sales-Search Model Results

In Figure 5.2 we report the posterior distribution of the number of regressors included in the model, that is q_γ. The average number of regressors included in the model is around 17 with a minimum near 5 and a maximum of 35 regressors. If the regressors were uniformly distributed among diffusion periods this would mean an average of 4 regressors per diffusion period.[4]

In Figure 5.3, 5.4 and 5.5 we graphically report the posterior means of the inclusion probabilities for all US states and for the Nintendo Wii, the Sony PS3 and the Xbox 360, respectively. All these probabilities are also reported in Table 5.2, 5.3 and 5.4. In Figures 5.3, 5.4 and 5.5 the lighter (green) colors represent high posterior probabilities while the darker (red) colors represent low inclusion probabilities. We include a map of the USA including state names in Figure 5.17 to facilitate the reading of these figures.

In Figure 5.3 we can observe that the states with the higher inclusion probabilities during the first diffusion period of the Nintendo Wii are Washington, Texas, Alabama, Wyoming, Kansas and New Hampshire. So, this means that these states are more likely to drive the sales of the Wii at an early stage of the Wii's life-cycle. It is noticeable too that the Western states are more likely to be included in the first diffusion period while the North-Eastern states have very low probability of inclusion. However, during the third diffusion period the Western states are not likely to be included in the model while it is more likely to include states in the center and North-East of the US. In the last diffusion period we find that very few states have high probabilities and these are Montana, North Dakota and New Hampshire. That is, there are many locations driving the growth of the Wii at early life-cycle stages and relatively few engines of growth at the end.

[4]Note that we chose $v_1 = 7$ and $a = 50$ and $b = 100$ (the parameters of the distribution of the prior inclusion probability w, see equations (5.9) and (5.10)) and this set-up results in a relatively small number of selected regressors q_γ.

The geographical pattern for the Sony PS3 is slightly similar to the pattern of the Nintendo Wii. However, we find that during the first diffusion period there are many more states (relative to the Wii) with high probability of inclusion. Again, all states in the West (California, Nevada, Oregon and Washington) have higher inclusion probabilities but for the PS3 many states in the East and North-East also have high probabilities during this first period. In fact, there are very few states with low probability of inclusion during the first period and these are North and South Dakota and Minnesota together with Kentucky and West Virginia. The opposite happens during the last diffusion period where many states have low probability of being included in the model. The probability of the West Coast states is high at the beginning and their influence seems to diminish in subsequent periods. The maps seem to be revealing a *boom bust* pattern. That is, many states may be influential during the first diffusion period but of this first set of countries very few remain influential in the last diffusion period and other states take the influential position.

The geographical pattern for the Microsoft Xbox 360 is very different from the other two consoles. The states with higher probabilities at each diffusion period are fewer than for the other two consoles and the influential states seem to be far from each other. However, for all regions, with the exception of Washington and Oregon, the states that seem more likely to be included in the model are in the North and North-East of the US.

An immediate question about these results is whether there is evidence of geographical clusters. At first glance, influential regions seem to be neighbors of other influential regions while not influential regions seem to be clustered together too. However, we may have some bias when judging probability distributions (Kahneman et al., 1982, page 32) and therefore we need some formal way to measure spatial association. Two statistics that can measure spatial association in aereal data are the Moran's I and the Geary's C (Banerjee et al., 2004, page 71).

We computed both the Moran's I and Geary's C for all sub-periods and technologies and we compared these two statistics, computed with the estimated inclusion probabilities, against the distribution of these two statistics when we assume that the probability of inclusion is uniformly distributed. Garber et al. (2004) also compare the observed spatial distribution of adoption against the uniform distribution. High spatial association is indicated by high Moran's I or by low Geary's C statistics. In Figure 5.6 we report the statistics computed with the real inclusion probabilities (in vertical dashed lines) and the distribution of both statistics (in the histograms) assuming the inclusion probabilities follow a uniform distribution.[5] As we can observe in Figure 5.6, when the inclusion probabilities are uniformly distributed the chances are very low to obtain the statistics in the extremes where the Moran's I and Geary's C based on the estimated inclusion probabilities appear. In the next section we discuss the results regarding the spatial model (equation (5.3)) where we further investigate the significance of the spatial clustering.

In the left panel of Figure 5.7, 5.8 and 5.9 we report the histogram of the posterior mean of the β coefficients for all sub-periods of the Nintendo Wii, the PS3 and Xbox 360, respectively. We report the distribution of the $\beta|\gamma = 1$ coefficients. That is, we report their distribution given that their corresponding regressor was included in the selected subset of regressors X_γ and we refer to these coefficients simply as β. In the right hand panel of the same figures we report the distribution of the posterior mean of the β coefficients divided by their posterior standard deviation. As we can see, the size of the β coefficients seems to be centered around 0.15 for the Nintendo Wii and the Xbox 360 and around 0.12 for the PS3. This means that on average a local (state) increase of 10% in search translates into a 1.5% or 1.2% increase in the global (nation) sales. The significance of the β coefficients varies from 1 up to 2 and there are approximately 25 regressors with a ratio (posterior mean over posterior standard deviation) higher than 1.5 and this number is quite satisfactory for a model with an average number of 17 regressors included.

In Figures 5.3, 5.4 and 5.5 we noticed that the probability of inclusion of different regions varies depending on the time period. In Figures 5.10, 5.11 and 5.12 we draw a scatter plot between the posterior mean of the search elasticity (the β coefficients) for each state and their probability of inclusion for the Nintendo Wii, the PS3 and Xbox 360, respectively. The vertical and horizontal lines correspond with the average inclusion probability and the average search elasticity, respectively. What we see in all three figures is that the place where states appear varies not only relative to their inclusion probabilities but also relative to the search elasticities. For example, in Figure 5.10 we see that the states with above average search elasticity and above

[5]We assume that the inclusion probability of each state is independent and identically distributed from other states and they follow a uniform with range [0,1]. We draw the probability for every state from the uniform and then we compute the Moran's I and Geary's C for L number of draws to obtain the probability distribution of these two statistics.

average inclusion probability (upper right quadrant) during the first period are Kansas, New Hampshire, Delaware, New Mexico, Nebraska, Arizona, New Jersey and California. However, the upper right quadrant states that appear in the following periods are different. For example, during the forth period the upper-quadrant states are North Dakota, Montana, Maine and New Hampshire. The Figures 5.11 and 5.12 for the PS3 and Xbox 360 confirm the same pattern, different groups of states appear at each quadrant of the scatter plots at each sub-period. These results point that some states may be important earlier in the diffusion of a technology while other states become important during later states of the diffusion. Note that this result is not explained by different introduction dates as the three consoles were launched simultaneously in all US states.

The sales-search model takes into account the relationship between aggregate sales and the online search at many different locations. This provides with interesting inclusion probabilities and we can rank the states according to their power to drive the aggregate sales. If we were to ignore all these details and we run a simple regression between aggregate sales and aggregate online search we obtain the results reported in Table 5.13. The overall sensitivity of sales to aggregate search (an indicator of search for all US) is larger than the sensitivity of sales to state-specific search. The estimates range from 0.17 up to 0.46, see the coefficient of search in this table. These last results seem intuitive but we miss the detailed region-specific analysis and a possible spatial story behind the results of the sales search model.

5.5.2 Results of the Spatial Model

In Table 5.5 we report the posterior mean and the posterior standard deviation of the δ coefficients of the spatial model (5.3). In the Table we report the δ coefficients for a set of seven variables. We tested other demographic variables measuring the ethnic origin and age distribution but we did not find them as significant and they were highly correlated with the set of seven variables that we kept in the model.

As we can observe, our results indicate that there is not a very strong association between demographic variables and the inclusion probabilities at each state. The reason why the posterior standard deviations might be large is because we have only 48 states in the probability model and therefore we have very few observations to estimate the coefficients. A second reason may be that we observe a relatively small variation in our dependent variable. Nonetheless, we find some interesting features in the δ_n coefficients.

The variables that seem to be relevant are the percentage of the population in college dorms and the percentage of the population that is married (percentage of households with married couples). Both of these variables are somewhat significant during the first and second diffusion periods. The effect of travel time to work is not significant but it is most of the time negative, as we would expect given than longer commuting time reduces leisure time to play video games or to search for consoles. Population density and income per capita seem to be slightly more important in the last diffusion stage while in the first stages of diffusion they are not. A last important feature to notice is that in many cases the size and sign of the δ_n coefficients may vary according to the diffusion stage of the products. For example, it may be that students and married couples tend to buy more video-game consoles at an early stage, as a high proportion of these groups increases the chance of a state being influential, while these groups may not buy at the end of the diffusion when we see that other parameters like population density and income per capita are slightly more important.

We estimate the spatial random effects Φ_n along side with the δ_n coefficients and we report their posterior mean and their posterior mean divided by their posterior standard deviation in Tables 5.6, 5.7, 5.8 and 5.9 for the first, second, third and fourth diffusion periods, respectively. In contrast with the δ_n coefficients, several of the spatial effects are significant. For example, in Table 5.6 we see that the spatial effect of Texas is significant both for the Nintendo Wii and the PS3 while it is not for the Xbox 360. This means that Texas is more likely to be driving the sales of the Wii and PS3 relative to the Xbox 360 during the first diffusion period. In the same table we notice that Ohio, South Dakota and Washington are positive and significant for the Xbox 360. The spatial effect of Washington is significant for all three technologies. Tables 5.7, 5.8 and 5.9 show similar many significant spatial effects during the rest of the diffusion periods.

In Figures 5.13, 5.14, 5.15 and 5.16 we report the distribution of the spatial effects for the Nintendo Wii and the first, second, third and fourth diffusion periods, respectively. In Figure 5.13 we can observe that for the first diffusion period the states with higher posterior spatial effects are Alabama, Delaware, Kentucky, Texas, Washington and Wyoming. The states with the lowest spatial effects are Georgia, Massachusetts, Missouri and Rhode Island. Texas and Wyoming continue to have a high spatial effect in the next diffusion period, see Figure 5.14 but the other states that had a high spatial effect in the first period no longer continue to be high in the second. In general, the spatial effect for each state varies according to the diffusion time of the technologies. For example, according to our results Texas is very influential for the Nintendo Wii at an early stage of its life-cycle while this state is not influential at the end of the life-cycle of the Wii.

We are finding significant spatial random effects for several states and all diffusion periods. However, a natural concern is whether the δ_n coefficients may have a different level of significance if we were to exclude the spatial effects from equation (5.3). In Table 5.10 we report the same δ_n coefficients estimated with ordinary least squares and their level of significance is relatively the same as before. Again, the population in college dorms and the percentage of households with married couples seem to be the more important variables. That is, the spatial effects explain geographical variation without affecting the inference we draw from the posteriors of the δ_n coefficients.

In Table 5.11 we present the posterior distribution of the correlations derived from the matrix Λ. The matrix Λ is a 3×3 covariance matrix and it measures the covariance between the spatial effects of different technologies. In the first diffusion period, for example, we find that the correlation of the spatial effects of the Xbox 360 are negatively correlated with the spatial effects of the PS3. The posterior mean of the correlation is -0.257 and the association is significant (zero is almost excluded from the 95% highest posterior density region). This negative correlation implies that if a state is likely to drive the sales of the Xbox 360 then it is not likely to drive the sales of the PS3. The association between the spatial effects of the Wii and those of the Xbox and PS3 are not different from zero (in these cases 0 is almost in the middle of the highest posterior density region) during the first diffusion period. We find some other significant associations during the third and fourth periods while in the second period we find no association between the spatial effects of the different technologies. The variation in correlation structure shows that at an early stage there is some degree of competition only between the

PS3 and Xbox 360 (because of the negative correlation in their spatial effects) while at later stages technologies seem to nurture each other (because we find significant positive correlations in their spatial effects).

Finally, in Table 5.12 we report the highest posterior density region for the ρ coefficients. We find roughly the same spatial decay (or spatial correlation) during all diffusion periods. The posterior mean of the ρ_n for all n is around 0.82. This number should be between 0 and 1 and numbers close to 1 indicate high spatial correlation between a state and its neighbors. The estimate of the ρ coefficient together with the Φ_n spatial effects are evidence of significant clusters of spillover effects between states. We do not know the direction of influence between the states but the model parameters capture significant spatial dependence among neighboring states.

5.6 Conclusions

We applied Bayesian variable selection methods to identify the influential locations for the diffusion of new technologies. We define influential locations as those that are more likely to drive the aggregate sales of the technologies. For our particular data on game consoles, we find that the influential locations change over time and that there is geographical clustering that is significantly captured by the spatial random effects in the probability model and by different measures of spatial association.

Moreover, we find variation in the groups of influential locations over time and the size of their associated search elasticity varies over time too. The search elasticity for the technologies at influential locations is on average 0.15. That is, an increase of 10% in local (state) search translates into a 1.5% increase in country level sales. Finally, we find some evidence of time variation in the association between spatial affects. Our results suggest that the geographical clustering is not driven by demographic heterogeneity and we find some evidence that suggests that the demographic effects vary over time.

In summary, our results suggest that influential locations may change over time together with the relationship between technologies and the relevance of demographics. The main managerial implications of this research is the notion that the group of influential locations is not fixed and therefore when a manager is looking to identify influentials, she or he should expect influentials to play a role at different locations and at different times. If managers were to ignore the spatial heterogeneity they will miss the valuable insights of how to allocate their marketing efforts based on the important locations for their products. The relevant question is not only who is influential but where and when and for how long a consumer or a group of consumers is influential.

5.7 Tables and Figures

```
library(RCurl)
library(XML)
wii_sales<-rep(0,416)
week.numbers<-seq((39838)-2184,40358,by=7)
for(i in 1:416)
{
part1<-"http://vgchartz.com/hwtable.php?cons[]=Wii&reg[]=America&start="
part2<-"&end="
week<-week.numbers[i]
url.dir<-paste(part1,week,part2,week,sep="")
url.text <- getURL(url.dir)
doc <- htmlParse(url.text,useInternalNodes=TRUE, error=function(...){})
x = xpathSApply(doc, "//table//td|//table//th", xmlValue)
wii_sales[i]<-as.numeric(gsub(",", ".", x[12]))
}
write.csv(wii_sales,file="wii_data.csv")
```

Note that the keyword `Wii` should be changed to `PS3` or `X360`
to retrieve the data for each of these consoles.

Table 5.1: R Code to Retrieve Data from VGChartz.com

	Posterior Inclusion Probabilities			
	1st Period	2nd Period	3rd Period	4th Period
Alabama	**0.111**	0.083	0.080	**0.117**
Alaska	0.074	0.069	0.091	**0.115**
Arizona	0.092	**0.110**	0.070	0.075
Arkansas	0.091	0.093	**0.103**	0.089
California	0.093	0.093	0.081	**0.116**
Colorado	0.076	0.069	0.098	0.079
Connecticut	0.074	0.057	**0.103**	0.071
Delaware	**0.105**	0.058	0.079	**0.103**
District of Columbia	0.076	**0.108**	0.077	0.083
Florida	0.096	0.061	0.090	0.077
Georgia	0.056	0.079	0.089	0.076
Hawaii	0.072	0.096	0.077	0.099
Idaho	0.086	0.082	0.075	**0.112**
Illinois	0.073	0.092	0.080	0.066
Indiana	0.079	0.059	0.065	0.082
Iowa	0.077	0.077	0.083	**0.125**
Kansas	**0.108**	0.085	0.088	0.083
Kentucky	0.075	0.091	0.093	0.099
Louisiana	**0.102**	**0.122**	0.081	0.065
Maine	0.079	0.080	0.090	**0.137**
Maryland	0.059	0.088	0.057	0.079
Massachusetts	0.084	**0.119**	0.096	0.074
Michigan	0.070	0.079	0.086	0.086
Minnesota	0.078	0.098	0.088	0.074
Mississippi	0.058	0.092	**0.105**	0.060
Missouri	0.086	0.075	0.088	0.093
Montana	0.095	0.084	0.099	**0.173**
Nebraska	0.092	0.073	0.093	0.090
Nevada	0.096	0.096	0.068	0.094
New Hampshire	**0.105**	0.097	0.076	**0.154**
New Jersey	0.095	**0.127**	**0.103**	0.073
New Mexico	0.099	0.096	**0.113**	**0.105**
New York	0.078	0.068	0.080	0.054
North Carolina	0.096	0.071	0.083	0.066
North Dakota	0.081	0.086	0.082	**0.190**
Ohio	0.078	0.090	**0.102**	0.089
Oklahoma	0.082	0.098	0.081	0.078
Oregon	0.098	**0.144**	0.063	0.055
Pennsylvania	0.064	0.081	0.065	0.062
Rhode Island	0.086	0.074	0.082	**0.101**
South Carolina	0.090	0.075	0.083	0.097
South Dakota	0.098	0.079	0.070	0.098
Tennessee	0.092	0.073	**0.119**	0.068
Texas	**0.129**	0.075	0.086	0.094
Utah	0.097	0.097	0.089	0.097
Vermont	0.076	0.073	**0.136**	0.091
Virginia	**0.100**	0.070	0.079	0.086
Washington	**0.126**	0.073	0.065	0.095
West Virginia	0.073	0.062	**0.108**	**0.119**
Wisconsin	0.072	0.076	**0.131**	0.060
Wyoming	**0.107**	**0.115**	**0.107**	**0.119**

Note: In bold probabilities larger than 0.10

Table 5.2: State Inclusion Probabilities for Each Diffusion Period for the Nintendo Wii

	Posterior Inclusion Probabilities			
	1st Period	2nd Period	3rd Period	4th Period
Alabama	0.088	0.073	0.094	0.086
Alaska	0.081	0.094	0.084	**0.185**
Arizona	0.081	0.063	0.091	0.057
Arkansas	**0.101**	0.090	0.098	0.093
California	0.096	0.096	0.088	0.080
Colorado	**0.106**	0.092	0.092	0.099
Connecticut	**0.104**	**0.102**	0.093	0.076
Delaware	0.086	0.078	**0.118**	0.090
District of Columbia	0.088	0.099	0.098	0.075
Florida	**0.100**	0.079	0.091	0.083
Georgia	0.095	0.097	**0.103**	0.067
Hawaii	0.098	0.080	0.094	0.088
Idaho	0.092	0.085	0.080	0.076
Illinois	0.091	**0.107**	0.082	0.088
Indiana	0.085	0.081	**0.104**	0.085
Iowa	0.087	**0.102**	0.087	0.093
Kansas	0.079	0.094	0.083	0.080
Kentucky	0.070	0.098	0.084	0.085
Louisiana	0.087	0.093	0.091	0.076
Maine	0.086	0.071	0.073	**0.115**
Maryland	0.095	**0.119**	0.085	0.093
Massachusetts	0.095	0.093	0.082	0.071
Michigan	0.089	**0.109**	0.081	0.086
Minnesota	0.073	0.068	0.081	0.086
Mississippi	0.086	0.085	0.087	0.078
Missouri	0.084	0.093	0.087	0.084
Montana	0.091	0.089	0.089	**0.103**
Nebraska	0.092	**0.109**	0.089	0.093
Nevada	0.096	0.087	0.090	0.072
New Hampshire	0.091	0.087	0.090	**0.140**
New Jersey	0.090	0.094	0.072	0.071
New Mexico	0.083	0.097	0.069	**0.105**
New York	0.096	0.089	0.093	0.064
North Carolina	**0.103**	0.083	0.082	0.071
North Dakota	0.070	0.076	0.084	0.094
Ohio	0.088	0.097	**0.105**	0.073
Oklahoma	0.080	0.091	0.091	0.084
Oregon	**0.104**	0.077	**0.101**	**0.102**
Pennsylvania	0.089	0.091	0.079	0.074
Rhode Island	0.081	0.087	0.082	**0.130**
South Carolina	0.090	0.092	0.076	0.090
South Dakota	0.066	0.068	0.079	0.094
Tennessee	0.089	0.087	0.095	0.091
Texas	**0.108**	0.093	**0.113**	0.065
Utah	**0.101**	0.072	**0.109**	0.097
Vermont	0.090	0.086	**0.100**	**0.141**
Virginia	0.096	0.083	0.062	0.073
Washington	**0.101**	0.081	0.095	0.069
West Virginia	0.074	0.083	**0.106**	0.097
Wisconsin	0.089	0.087	0.092	0.095
Wyoming	0.092	0.086	0.094	0.090

Note: In bold probabilities larger than 0.10

Table 5.3: State Inclusion Probabilities for Each Diffusion Period for the Sony PS3

	Posterior Inclusion Probabilities			
	1st Period	2nd Period	3rd Period	4th Period
Alabama	0.085	0.091	0.090	0.079
Alaska	**0.104**	**0.188**	0.080	**0.199**
Arizona	0.077	0.074	0.080	0.054
Arkansas	0.098	0.082	0.087	0.075
California	0.099	0.082	0.075	0.074
Colorado	0.078	0.088	0.087	0.084
Connecticut	0.078	0.081	0.091	**0.101**
Delaware	**0.116**	**0.136**	0.075	**0.204**
District of Columbia	0.091	0.096	0.097	0.071
Florida	**0.102**	0.066	**0.100**	0.065
Georgia	0.084	0.073	**0.115**	0.092
Hawaii	0.089	**0.115**	0.055	0.076
Idaho	0.087	**0.109**	0.076	**0.137**
Illinois	0.086	0.075	**0.105**	**0.100**
Indiana	0.086	0.087	0.074	0.064
Iowa	0.097	**0.126**	0.083	**0.100**
Kansas	**0.114**	0.082	0.087	0.081
Kentucky	**0.103**	0.078	**0.101**	**0.109**
Louisiana	0.067	0.074	0.082	0.058
Maine	0.097	**0.113**	0.097	0.095
Maryland	0.087	0.066	0.085	0.087
Massachusetts	0.095	**0.100**	0.085	0.079
Michigan	0.092	0.076	0.096	0.082
Minnesota	0.097	0.092	0.073	0.095
Mississippi	0.096	0.062	**0.131**	0.080
Missouri	0.079	0.087	0.098	0.073
Montana	0.071	0.059	0.087	0.096
Nebraska	0.084	0.067	0.071	0.095
Nevada	0.093	0.074	0.071	0.084
New Hampshire	0.089	0.098	0.089	**0.119**
New Jersey	0.085	**0.110**	0.095	0.071
New Mexico	0.091	**0.112**	0.071	**0.100**
New York	0.083	**0.106**	**0.101**	0.093
North Carolina	0.091	**0.103**	0.090	0.066
North Dakota	**0.129**	0.082	**0.113**	**0.113**
Ohio	0.099	0.094	0.083	0.079
Oklahoma	0.086	0.085	0.084	0.094
Oregon	**0.116**	0.081	0.087	0.081
Pennsylvania	0.096	0.087	0.093	0.085
Rhode Island	**0.102**	**0.113**	0.092	**0.127**
South Carolina	0.090	0.081	0.082	0.073
South Dakota	**0.132**	0.097	**0.118**	**0.102**
Tennessee	0.084	0.096	**0.134**	0.089
Texas	0.093	0.073	0.078	0.063
Utah	0.085	0.088	0.077	0.059
Vermont	0.082	**0.110**	**0.152**	0.082
Virginia	**0.103**	0.074	0.078	0.064
Washington	**0.111**	**0.101**	**0.100**	0.079
West Virginia	0.085	0.065	**0.140**	**0.114**
Wisconsin	0.090	0.087	0.093	0.086
Wyoming	0.070	**0.125**	**0.105**	0.084

Note: In bold probabilities larger than 0.10

Table 5.4: State Inclusion Probabilities for Each Diffusion Period for the Microsoft Xbox 360

	MCAR First Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3684	0.0163	-145.2131
Male Female Ratio	0.0103	0.0326	0.3152
Population Density	0.0042	0.0270	0.1569
Population in College Dorms	0.0337	0.0200	1.6820
Married Couple	0.0236	0.0171	1.3834
Travel Time to Work	-0.0015	0.0194	-0.0751
Income per Capita	0.0106	0.0157	0.6723

	MCAR Second Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.4029	0.0177	-135.4291
Male Female Ratio	0.0345	0.0361	0.9561
Population Density	0.0357	0.0285	1.2506
Population in College Dorms	0.0304	0.0232	1.3138
Married Couple	-0.0208	0.0202	-1.0332
Travel Time to Work	-0.0183	0.0221	-0.8307
Income per Capita	-0.0185	0.0185	-1.0007

	MCAR Third Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3715	0.0214	-110.8737
Male Female Ratio	-0.0192	0.0409	-0.4694
Population Density	-0.0294	0.0343	-0.8562
Population in College Dorms	-0.0245	0.0251	-0.9758
Married Couple	0.0231	0.0251	0.9208
Travel Time to Work	0.0163	0.0258	0.6329
Income per Capita	-0.0103	0.0199	-0.5181

	MCAR Fourth Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3987	0.0283	-84.6213
Male Female Ratio	-0.0305	0.0589	-0.5171
Population Density	0.0464	0.0474	0.9795
Population in College Dorms	-0.0218	0.0339	-0.6436
Married Couple	-0.0013	0.0324	-0.0402
Travel Time to Work	-0.0157	0.0352	-0.4457
Income per Capita	0.0191	0.0267	0.7141

Note: The first column reports the posterior mean of the coefficient. The second column reports the posterior standard deviation and the third column reports the ratio of the posterior mean over the posterior standard deviation, called here t-value.

Table 5.5: Posterior of MCAR δ coefficients

	MCAR First Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	0.260	3.990	0.025	0.380	-0.011	-0.196
Arizona	0.053	0.780	-0.066	-1.203	-0.131	-2.336
Arkansas	0.041	0.770	0.141	2.329	0.110	2.093
California	-0.038	-0.481	-0.001	-0.035	0.023	0.356
Colorado	-0.164	-2.412	0.154	2.322	-0.141	-2.268
Connecticut	-0.213	-2.762	0.135	1.903	-0.155	-2.167
Delaware	0.201	3.464	-0.005	-0.103	0.302	5.033
Florida	0.081	1.306	0.132	2.304	0.150	2.406
Georgia	-0.462	-6.174	0.063	1.322	-0.055	-1.172
Idaho	-0.243	-3.420	0.053	0.952	-0.034	-0.525
Illinois	-0.070	-1.349	-0.007	-0.261	-0.053	-1.149
Indiana	-0.232	-4.463	-0.014	-0.237	-0.066	-1.417
Iowa	-0.138	-2.370	-0.058	-1.276	-0.051	-0.861
Kansas	-0.162	-3.003	-0.037	-0.728	0.074	1.578
Kentucky	0.205	3.492	-0.101	-2.010	0.255	4.228
Louisiana	-0.097	-1.705	-0.174	-2.826	0.218	3.517
Maine	0.183	2.815	0.010	0.151	-0.254	-4.285
Maryland	-0.099	-1.467	-0.020	-0.460	0.105	1.764
Massachusetts	-0.443	-5.272	0.027	0.278	-0.064	-0.882
Michigan	-0.078	-1.512	0.041	0.791	0.047	0.952
Minnesota	-0.264	-4.806	-0.037	-0.761	0.009	0.107
Mississippi	-0.068	-0.913	-0.137	-2.282	0.152	2.150
Missouri	-0.409	-5.690	-0.020	-0.524	0.080	1.520
Montana	-0.013	-0.253	-0.037	-0.700	-0.095	-1.832
Nebraska	0.069	1.258	0.026	0.548	-0.217	-3.822
Nevada	0.056	0.629	0.055	0.717	-0.030	-0.324
New Hampshire	0.060	0.952	0.060	1.082	0.027	0.455
New Jersey	0.134	1.442	-0.011	-0.131	-0.034	-0.400
New Mexico	0.129	2.338	0.066	1.228	0.012	0.143
New York	0.052	0.622	-0.116	-1.556	-0.029	-0.410
North Carolina	-0.160	-3.347	0.046	0.966	-0.099	-1.794
North Dakota	0.089	1.459	0.153	2.331	0.037	0.568
Ohio	-0.126	-2.172	-0.266	-3.950	0.342	5.245
Oklahoma	-0.125	-2.507	0.003	0.129	0.118	2.365
Oregon	-0.065	-1.420	-0.081	-1.685	-0.020	-0.365
Pennsylvania	0.027	0.433	0.075	1.114	0.187	2.729
Rhode Island	-0.289	-3.190	0.059	0.587	0.133	1.393
South Carolina	0.035	0.472	0.044	0.780	0.043	0.771
South Dakota	0.108	2.093	-0.276	-4.334	0.409	6.168
Tennessee	0.034	0.688	0.006	0.111	-0.054	-1.247
Texas	0.314	4.926	0.128	2.476	-0.020	-0.262
Utah	0.009	0.147	0.047	0.716	-0.124	-1.475
Vermont	-0.139	-2.786	0.037	0.498	-0.057	-1.164
Virginia	0.082	1.547	0.038	0.831	0.109	2.372
Washington	0.353	6.191	0.121	2.318	0.224	4.035
West Virginia	-0.160	-2.826	-0.142	-2.508	-0.011	-0.243
Wisconsin	-0.242	-4.815	-0.024	-0.626	-0.013	-0.288
Wyoming	0.180	2.908	0.033	0.611	-0.239	-3.781

Note: The numbers correspond to the Φ parameters of the MCAR model for the first diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 5.6: Posterior of MCAR Spatial Effects

	MCAR Second Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	0.005	0.082	-0.106	-1.394	0.096	1.139
Arizona	0.231	2.843	-0.278	-2.827	-0.148	-1.867
Arkansas	0.096	1.393	0.061	0.972	-0.024	-0.365
California	-0.004	-0.039	0.019	0.185	-0.134	-1.459
Colorado	-0.211	-2.324	0.053	0.683	0.005	0.091
Connecticut	-0.392	-4.157	0.172	1.713	-0.061	-0.599
Delaware	-0.365	-4.093	-0.080	-0.950	0.482	5.568
Florida	-0.350	-4.939	-0.090	-1.367	-0.284	-3.924
Georgia	-0.072	-1.085	0.118	1.746	-0.153	-2.014
Idaho	0.113	1.363	-0.057	-0.792	0.288	3.083
Illinois	-0.063	-0.859	-0.020	-0.334	0.217	2.850
Indiana	0.046	0.592	0.180	2.409	-0.152	-2.212
Iowa	-0.349	-4.061	-0.047	-0.740	0.017	0.274
Kansas	-0.100	-1.419	0.159	2.145	0.382	4.606
Kentucky	0.005	0.129	0.095	1.494	-0.030	-0.495
Louisiana	0.072	0.934	0.127	1.718	-0.089	-1.073
Maine	0.406	5.404	0.123	1.628	-0.104	-1.115
Maryland	-0.035	-0.398	-0.140	-1.670	0.305	3.512
Massachusetts	-0.054	-0.581	0.232	2.550	-0.321	-3.198
Michigan	0.306	4.063	0.053	0.796	0.129	1.833
Minnesota	-0.084	-1.101	0.223	2.738	-0.111	-1.645
Mississippi	0.139	1.745	-0.203	-2.384	0.085	1.070
Missouri	0.082	1.156	0.018	0.254	-0.278	-3.091
Montana	-0.163	-2.117	0.044	0.563	-0.017	-0.213
Nebraska	-0.020	-0.301	0.034	0.612	-0.351	-3.717
Nevada	-0.212	-1.893	0.161	1.571	-0.299	-2.607
New Hampshire	0.176	2.273	0.074	0.955	-0.086	-1.124
New Jersey	0.091	0.830	-0.013	-0.146	0.108	1.013
New Mexico	0.372	4.661	0.069	1.076	0.234	2.975
New York	0.029	0.272	0.032	0.391	0.182	1.950
North Carolina	-0.259	-3.558	0.010	0.128	0.177	2.434
North Dakota	-0.237	-2.899	-0.075	-1.059	0.134	1.736
Ohio	-0.036	-0.403	-0.137	-1.909	-0.087	-1.222
Oklahoma	0.052	0.714	0.102	1.671	0.087	1.135
Oregon	0.128	1.746	0.050	0.816	-0.012	-0.202
Pennsylvania	0.442	4.291	-0.143	-1.726	-0.105	-1.268
Rhode Island	-0.139	-1.234	-0.022	-0.172	-0.079	-0.691
South Carolina	-0.133	-1.663	0.080	1.223	-0.055	-0.710
South Dakota	-0.102	-1.486	-0.216	-2.726	0.096	1.267
Tennessee	-0.137	-1.954	0.027	0.502	0.108	1.570
Texas	-0.171	-2.121	0.043	0.518	-0.207	-2.642
Utah	0.160	1.554	-0.118	-1.298	0.078	0.766
Vermont	-0.137	-1.648	0.025	0.432	0.269	3.278
Virginia	-0.181	-2.476	-0.013	-0.137	-0.125	-1.923
Washington	-0.156	-1.911	-0.051	-0.760	0.164	1.966
West Virginia	-0.251	-2.873	0.018	0.230	-0.211	-2.454
Wisconsin	-0.143	-1.858	0.000	-0.016	-0.011	-0.187
Wyoming	0.254	2.904	-0.031	-0.453	0.333	3.692

Note: The numbers correspond to the Φ parameters of the MCAR model for the second diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 5.7: Posterior of MCAR Spatial Effects

	MCAR Third Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	-0.155	-2.445	0.003	0.000	-0.045	-0.739
Arizona	-0.247	-4.132	0.003	0.112	-0.122	-2.098
Arkansas	0.085	1.788	0.038	0.831	-0.075	-1.489
California	-0.016	-0.192	0.060	0.748	-0.093	-1.187
Colorado	0.111	1.571	0.056	0.795	0.002	0.000
Connecticut	0.216	2.797	0.107	1.416	0.093	1.206
Delaware	-0.120	-2.013	0.292	4.726	-0.169	-2.349
Florida	0.033	0.730	0.040	0.932	0.141	3.158
Georgia	-0.003	-0.161	0.134	2.776	0.256	5.227
Idaho	-0.209	-2.808	-0.011	-0.166	-0.548	-5.979
Illinois	-0.143	-3.097	-0.077	-1.455	-0.135	-2.773
Indiana	-0.104	-2.571	-0.069	-1.580	0.176	3.609
Iowa	-0.343	-5.740	0.131	2.517	-0.213	-3.606
Kansas	-0.089	-1.768	-0.039	-0.748	-0.090	-1.730
Kentucky	-0.055	-1.379	-0.114	-2.935	-0.068	-1.564
Louisiana	0.013	0.159	-0.083	-1.145	0.102	1.350
Maine	-0.147	-2.509	-0.026	-0.453	-0.136	-2.225
Maryland	0.029	0.381	-0.183	-2.747	0.101	1.594
Massachusetts	-0.336	-3.908	0.069	0.842	0.068	0.830
Michigan	0.107	2.373	-0.049	-1.242	-0.023	-0.579
Minnesota	-0.028	-0.632	-0.086	-1.840	0.081	1.874
Mississippi	-0.049	-0.737	-0.129	-1.942	-0.244	-3.254
Missouri	0.149	2.612	-0.041	-0.930	0.369	5.740
Montana	-0.035	-0.683	-0.039	-0.789	0.084	1.485
Nebraska	0.081	1.574	-0.028	-0.577	-0.045	-0.986
Nevada	0.074	0.727	0.034	0.368	-0.194	-1.727
New Hampshire	-0.316	-5.361	-0.027	-0.516	-0.273	-4.767
New Jersey	-0.079	-0.704	0.090	0.850	0.074	0.646
New Mexico	0.118	2.261	-0.233	-3.983	0.036	0.626
New York	0.340	3.580	-0.147	-1.705	-0.120	-1.397
North Carolina	-0.092	-2.102	0.059	1.387	0.142	3.221
North Dakota	-0.079	-1.263	-0.082	-1.350	0.007	0.153
Ohio	-0.046	-0.761	-0.010	-0.245	0.280	4.138
Oklahoma	0.106	2.612	0.127	3.062	-0.109	-2.295
Oregon	-0.098	-2.227	0.018	0.431	-0.061	-1.229
Pennsylvania	-0.286	-3.265	0.176	2.233	0.033	0.389
Rhode Island	-0.222	-1.949	-0.023	-0.192	0.144	1.246
South Carolina	-0.089	-1.493	-0.176	-2.974	-0.105	-2.070
South Dakota	-0.252	-4.378	-0.132	-2.261	0.261	4.083
Tennessee	0.276	5.944	0.047	1.130	0.396	6.843
Texas	-0.020	-0.361	0.250	4.013	-0.115	-1.882
Utah	-0.094	-0.951	0.098	0.941	-0.242	-2.204
Vermont	0.411	7.663	0.088	1.529	0.530	9.155
Virginia	-0.113	-2.047	-0.343	-6.009	-0.124	-2.607
Washington	-0.321	-5.615	0.069	1.359	0.119	2.074
West Virginia	0.117	1.873	0.100	1.595	0.386	5.353
Wisconsin	0.404	7.422	0.040	0.977	0.056	1.251
Wyoming	0.180	2.606	0.042	0.677	0.154	2.353

Note: The numbers correspond to the Φ parameters of the MCAR model for the third diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 5.8: Posterior of MCAR Spatial Effects

	MCAR Fourth Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	0.311	3.294	-0.008	-0.171	-0.092	-1.108
Arizona	-0.092	-1.076	-0.367	-4.305	-0.412	-4.807
Arkansas	0.044	0.613	0.078	1.217	-0.123	-1.864
California	0.377	3.187	-0.005	-0.083	-0.080	-0.759
Colorado	-0.040	-0.449	0.178	1.738	0.022	0.201
Connecticut	-0.386	-3.484	-0.306	-2.844	-0.017	-0.144
Delaware	0.079	0.999	-0.045	-0.631	0.806	9.027
Florida	-0.136	-2.313	-0.065	-1.059	-0.313	-5.411
Georgia	-0.103	-1.827	-0.228	-3.586	0.088	1.323
Idaho	0.197	1.842	0.072	0.719	-0.077	-0.745
Illinois	0.277	4.295	-0.109	-1.978	0.485	7.363
Indiana	-0.258	-4.541	0.022	0.413	0.158	3.015
Iowa	-0.063	-0.839	-0.021	-0.271	-0.293	-4.108
Kansas	0.395	6.189	0.079	1.320	0.156	2.558
Kentucky	-0.027	-0.567	-0.062	-1.127	-0.049	-0.780
Louisiana	0.155	1.626	-0.006	-0.162	0.254	2.682
Maine	-0.307	-3.621	-0.149	-1.864	-0.412	-4.844
Maryland	0.372	3.837	0.191	2.154	0.002	-0.065
Massachusetts	-0.233	-2.048	-0.065	-0.597	-0.140	-1.277
Michigan	-0.155	-2.323	-0.191	-3.465	-0.089	-1.414
Minnesota	0.006	0.061	0.007	0.110	-0.038	-0.671
Mississippi	-0.137	-1.491	0.007	-0.006	0.113	1.163
Missouri	-0.374	-4.820	-0.103	-1.523	-0.076	-1.138
Montana	0.118	1.436	0.006	0.001	-0.132	-1.815
Nebraska	0.729	8.505	0.187	2.936	0.118	1.865
Nevada	0.132	0.819	0.160	1.043	0.180	1.101
New Hampshire	0.066	0.846	-0.196	-2.587	-0.054	-0.703
New Jersey	0.394	2.581	0.285	1.962	0.124	0.840
New Mexico	-0.151	-2.031	-0.168	-2.577	-0.181	-2.489
New York	0.219	1.758	0.203	1.710	0.162	1.303
North Carolina	-0.462	-7.091	-0.277	-4.951	0.081	1.415
North Dakota	-0.247	-3.025	-0.166	-1.989	-0.251	-3.191
Ohio	0.805	8.115	0.075	0.965	0.260	3.472
Oklahoma	0.050	1.012	-0.146	-2.722	-0.071	-1.272
Oregon	-0.073	-1.372	-0.004	-0.084	0.109	1.615
Pennsylvania	-0.443	-3.824	0.165	1.630	-0.056	-0.544
Rhode Island	-0.566	-3.806	-0.380	-2.586	-0.243	-1.660
South Carolina	0.121	1.680	0.038	0.472	-0.175	-2.585
South Dakota	0.149	2.292	0.100	1.566	0.191	2.710
Tennessee	-0.235	-3.317	0.053	0.929	0.029	0.457
Texas	0.163	1.913	-0.198	-2.502	-0.239	-3.037
Utah	0.178	1.235	0.169	1.186	-0.324	-2.171
Vermont	0.052	0.736	0.489	6.011	-0.060	-0.965
Virginia	-0.011	-0.171	-0.166	-2.960	-0.304	-5.075
Washington	0.133	1.688	-0.188	-2.728	-0.060	-0.839
West Virginia	0.355	3.908	0.144	1.711	0.302	3.604
Wisconsin	-0.351	-5.133	0.099	1.862	0.008	0.121
Wyoming	0.375	3.677	0.088	0.941	0.018	0.213

Note: The numbers correspond to the Φ parameters of the MCAR model for the fourth diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 5.9: Posterior of MCAR Spatial Effects

	OLS First Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3730	0.0131	-180.7930
Male Female Ratio	0.0185	0.0195	0.9480
Population Density	0.0039	0.0228	0.1720
Population in College Dorms	0.0263	0.0164	1.6040
Married Couple	0.0198	0.0161	1.2250
Travel Time to Work	0.0131	0.0194	0.6740
Income per Capita	-0.0028	0.0217	-0.1300

	OLS Second Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.4053	0.0159	-151.0310
Male Female Ratio	0.0218	0.0237	0.9230
Population Density	0.0239	0.0277	0.8620
Population in College Dorms	0.0257	0.0199	1.2910
Married Couple	-0.0092	0.0196	-0.4710
Travel Time to Work	-0.0136	0.0235	-0.5780
Income per Capita	-0.0194	0.0263	-0.7390

	OLS Third Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3730	0.0131	-180.7930
Male Female Ratio	0.0185	0.0195	0.9480
Population Density	0.0039	0.0228	0.1720
Population in College Dorms	0.0263	0.0164	1.6040
Married Couple	0.0198	0.0161	1.2250
Travel Time to Work	0.0131	0.0194	0.6740
Income per Capita	-0.0028	0.0217	-0.1300

	OLS Fourth Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.4008	0.0203	-118.0540
Male Female Ratio	-0.0197	0.0302	-0.6520
Population Density	0.0355	0.0354	1.0030
Population in College Dorms	-0.0237	0.0254	-0.9310
Married Couple	0.0210	0.0250	0.8400
Travel Time to Work	0.0189	0.0300	0.6290
Income per Capita	0.0182	0.0336	0.5410

Note: These are parameter estimates of the model in equation (5.3) obtained by OLS and with no spatial effects.

Table 5.10: OLS δ coefficients

	MCAR First Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	0.075	-0.185	0.337
Λ_{13} (Wii-Xbox)	0.115	-0.139	0.352
Λ_{23} (PS3-Xbox)	-0.257	-0.488	0.036

	MCAR Second Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	-0.082	-0.344	0.210
Λ_{13} (Wii-Xbox)	0.096	-0.156	0.354
Λ_{23} (PS3-Xbox)	-0.103	-0.377	0.179

	MCAR Third Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	0.061	-0.204	0.302
Λ_{13} (Wii-Xbox)	0.401	0.145	0.600
Λ_{23} (PS3-Xbox)	0.117	-0.137	0.368

	MCAR Fourth Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	0.409	0.156	0.601
Λ_{13} (Wii-Xbox)	0.349	0.090	0.552
Λ_{23} (PS3-Xbox)	0.311	0.058	0.534

Note: We present the posterior mean and the posterior 95% highest density region of the correlation matrix obtained from the Λ matrix. The Λ matrix measures the covariance between the spatial effects of the three products.

Table 5.11: Posterior of MCAR Λ correlations

	HPDR		
	95%	50%	5%
MCAR 1st period ρ	0.975	0.805	0.150
MCAR 2nd period ρ	0.975	0.825	0.150
MCAR 3rd period ρ	0.975	0.825	0.150
MCAR 4th period ρ	0.975	0.815	0.200
Note:			

Table 5.12: Highest Posterior Density Region (HPDR) for the ρ coefficient.

	Aggregate Model for the Wii		
Variable	Estimate	Std. Error	t-value
spline	0.663	0.088	7.554
Search Wii	0.468	0.121	3.862

	Aggregate Model for the PS3		
Variable	Estimate	Std. Error	t-value
spline	0.862	0.108	7.974
Search PS3	0.171	0.133	1.287

	Aggregate Model for the X360		
Variable	Estimate	Std. Error	t-value
spline	0.722	0.122	5.916
Search X360	0.375	0.163	2.304

Note: The dependent variable is aggregate sales for each of the consoles (in logs). The right hand side includes a spline term and the logs of the search index for the console. The R^2 is higher than 0.95 for all three regressions.

Table 5.13: OLS Regressions between Aggregate Sales Data and Aggregate Online Search Data

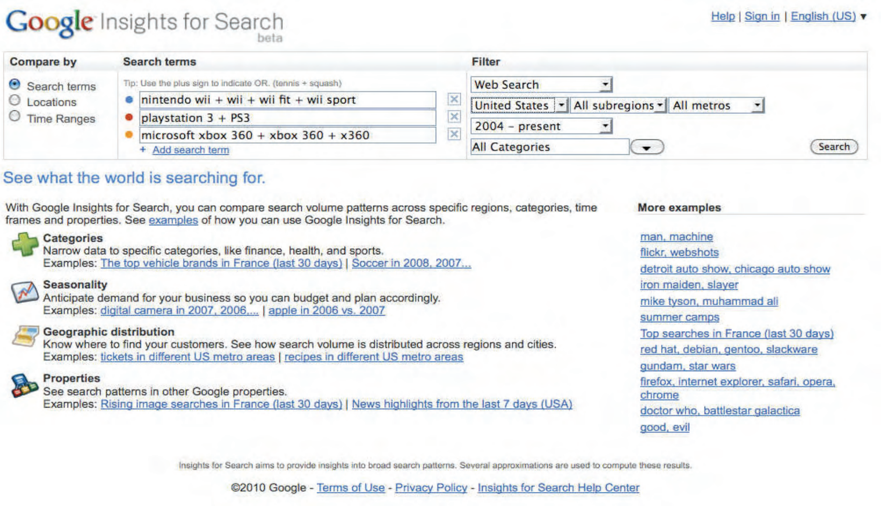

Figure 5.1: Google Insights for Search

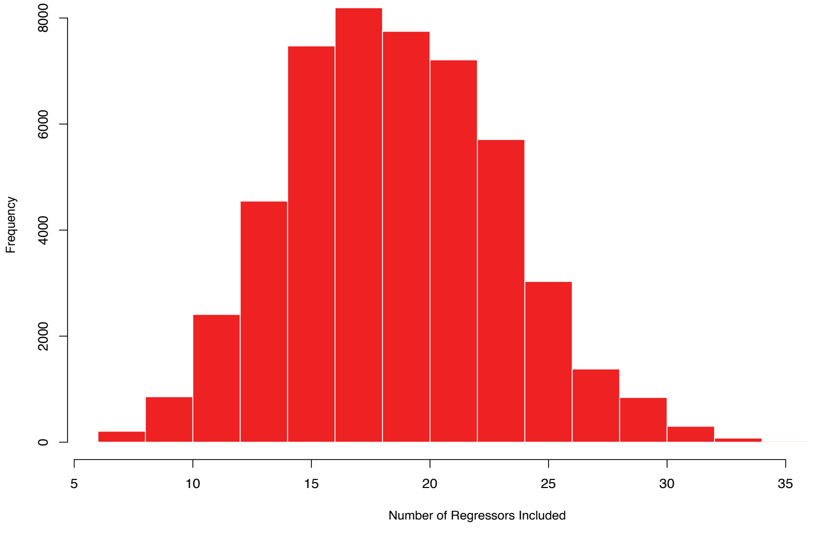

Figure 5.2: Model Size: Posterior Distribution of the Number of Regressors Included in the Model for the Nintendo Wii

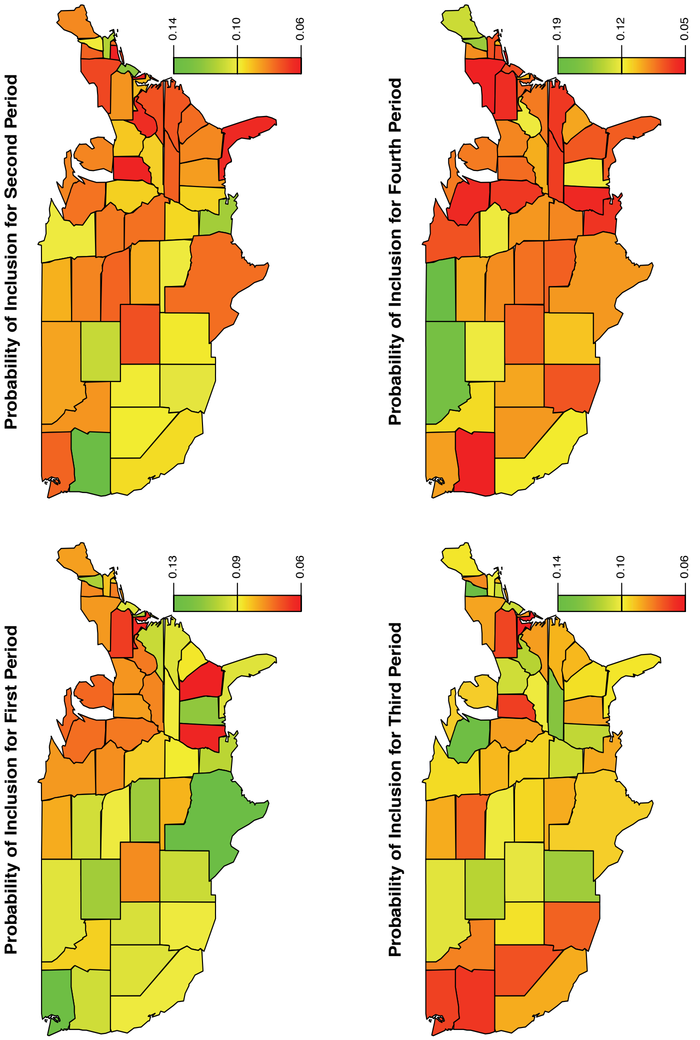

Figure 5.3: State Inclusion Probabilities for Each Diffusion Period of the Nintendo Wii

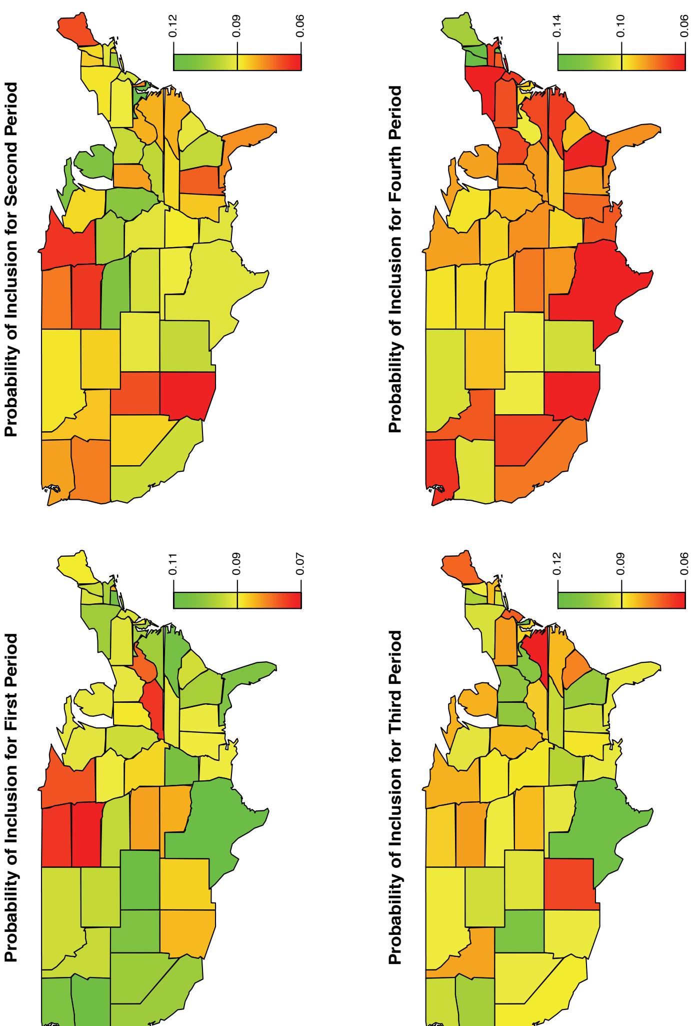

Figure 5.4: State Inclusion Probabilities for Each Diffusion Period of the Sony PS3

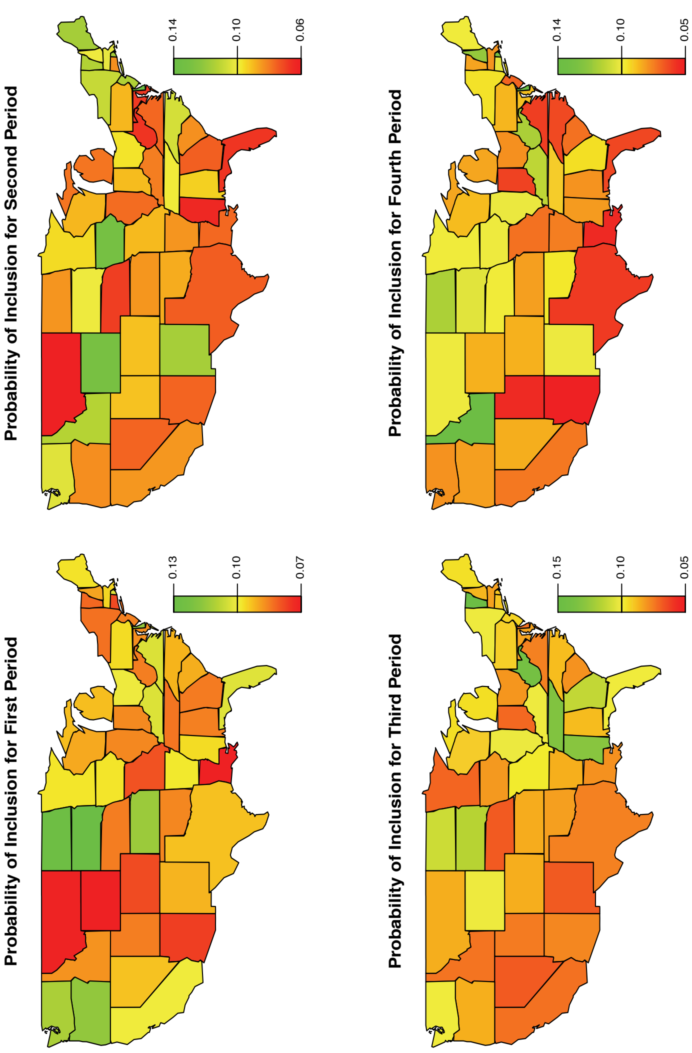

Figure 5.5: State Inclusion Probabilities for Each Diffusion Period of the Xbox 360

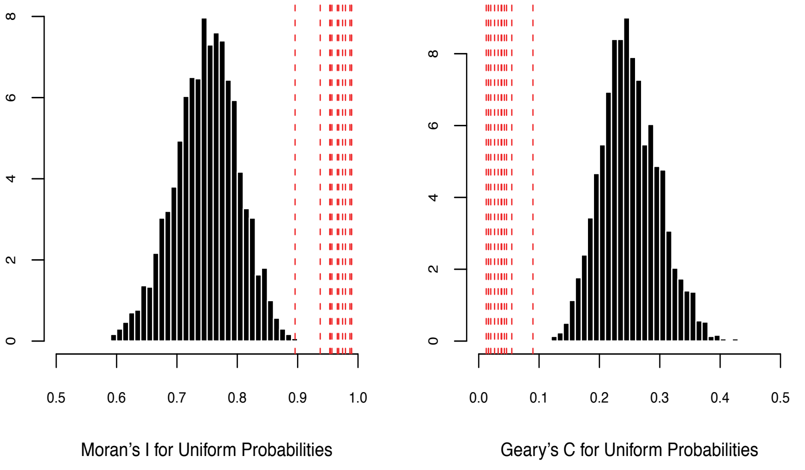

Figure 5.6: Moran's I and Geary's C for Uniform Probabilities (Histogram) and Moran's I and Geary's C for all Diffusion Periods and Technologies (Vertical Lines)

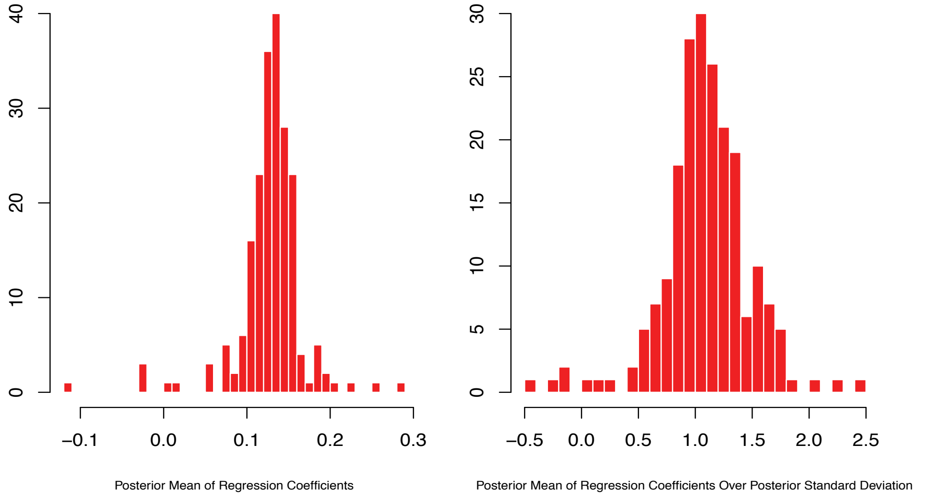

Figure 5.7: Nintendo Wii Model: Histogram of the Posterior Mean of the Regression Coefficient for all US States and All Time Periods (Left Panel) and Posterior Mean Over Posterior Standard Deviation (Right Panel)

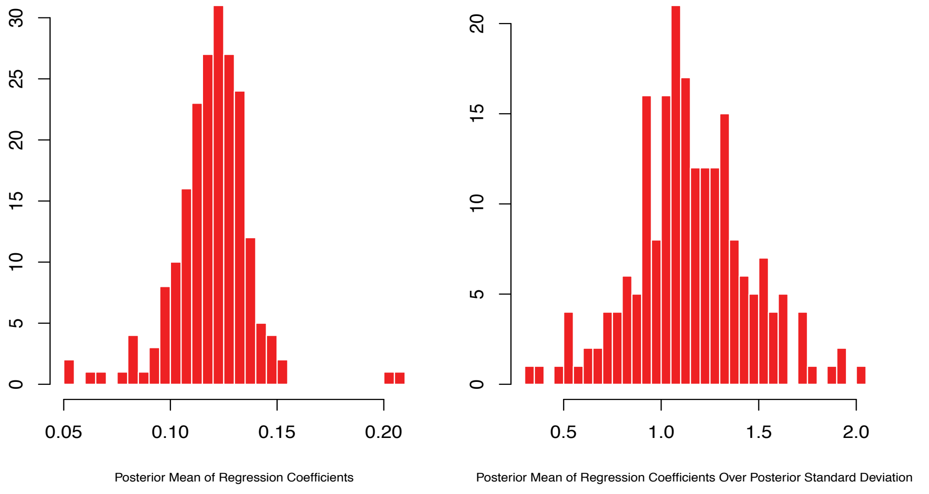

Figure 5.8: Sony PS3 Model: Histogram of the Posterior Mean of the Regression Coefficient for all US States and All Time Periods (Left Panel) and Posterior Mean Over Posterior Standard Deviation (Right Panel)

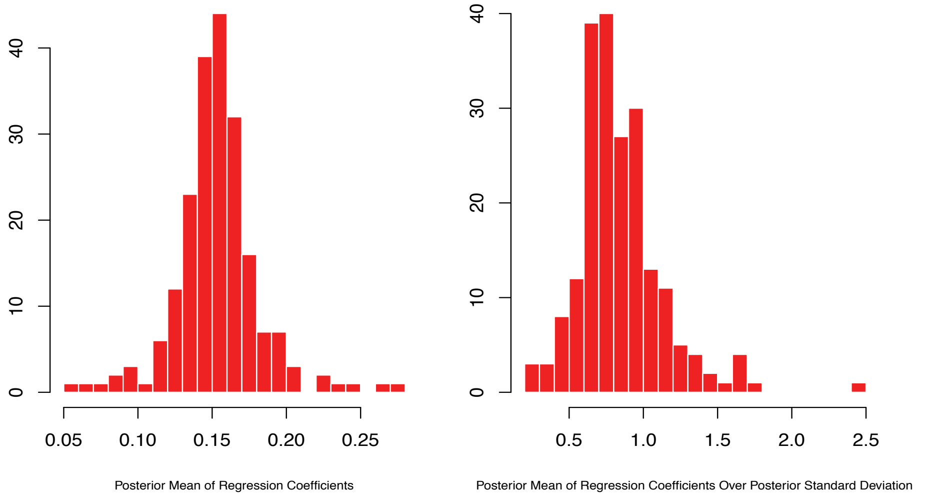

Figure 5.9: Microsoft Xbox Model: Histogram of the Posterior Mean of the Regression Coefficient for all US States and All Time Periods (Left Panel) and Posterior Mean Over Posterior Standard Deviation (Right Panel)

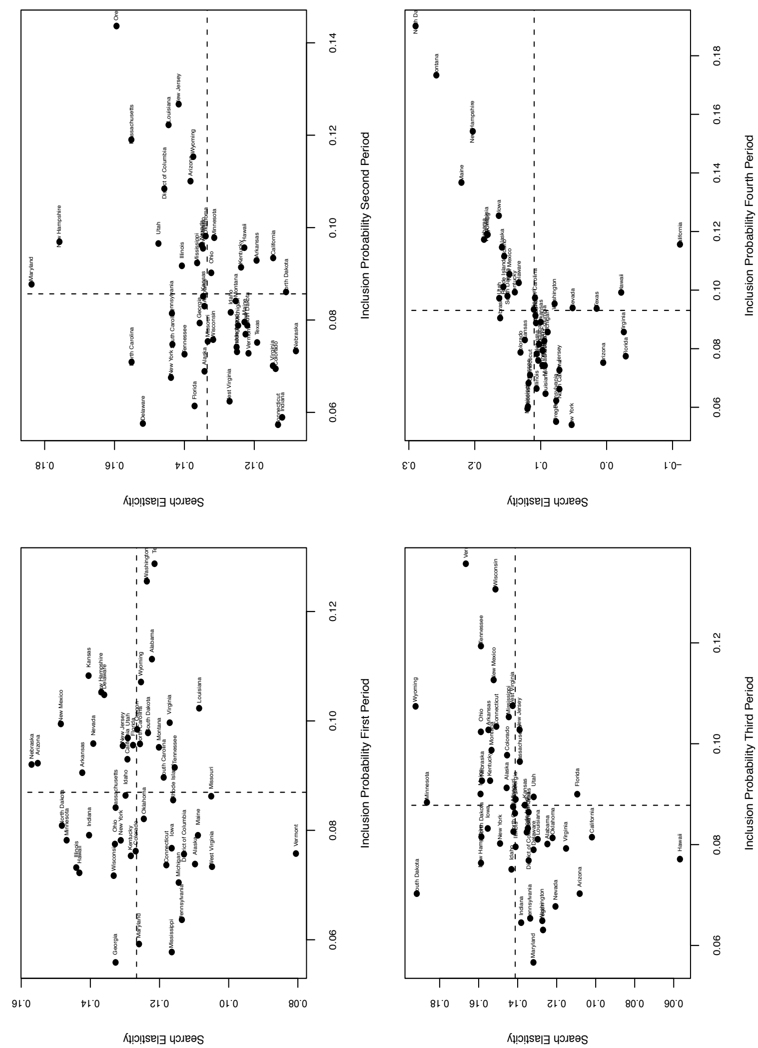

Figure 5.10: Scatter Plots between Inclusion Probabilities for Each Diffusion Period and Search Elasticity (Nintendo Wii)

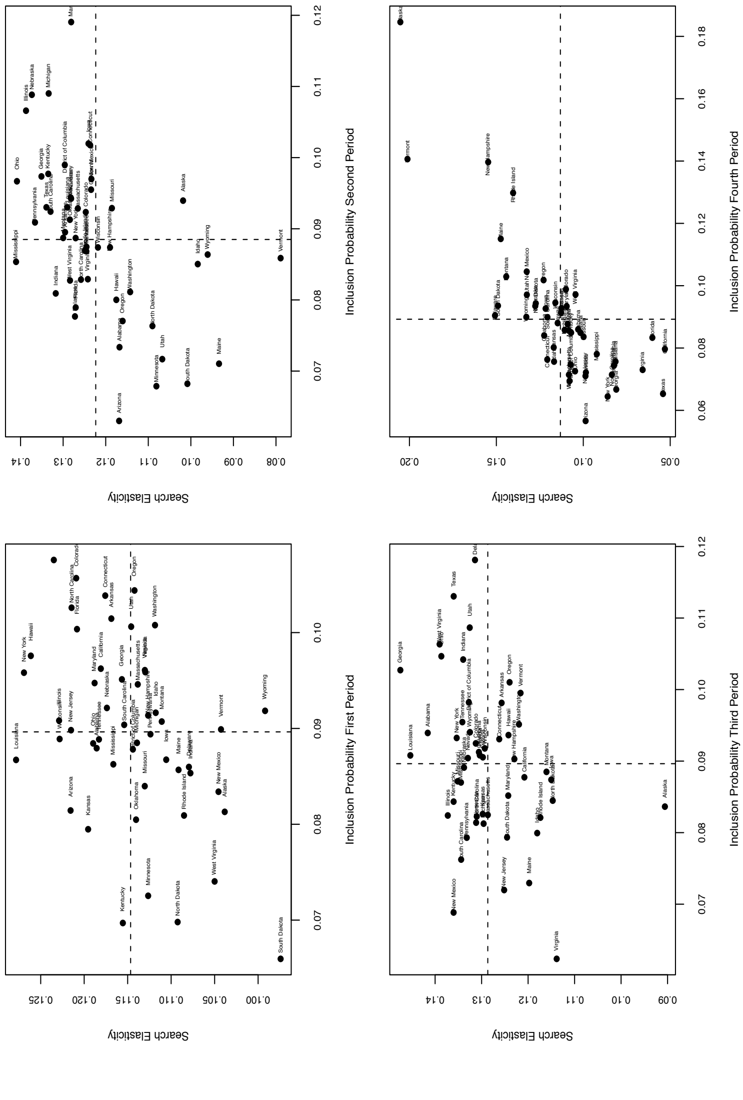

Figure 5.11: Scatter Plots between Inclusion Probabilities for Each Diffusion Period and Search Elasticity (Sony PS3)

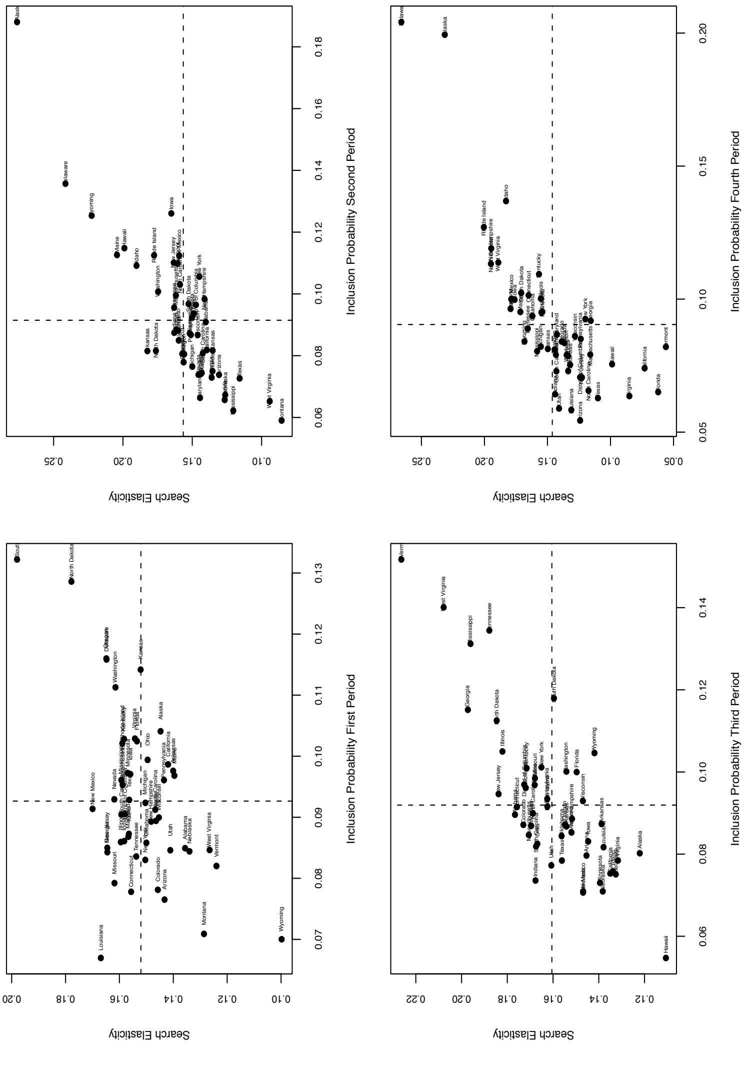

Figure 5.12: Scatter Plots between Inclusion Probabilities for Each Diffusion Period and Search Elasticity (Xbox 360)

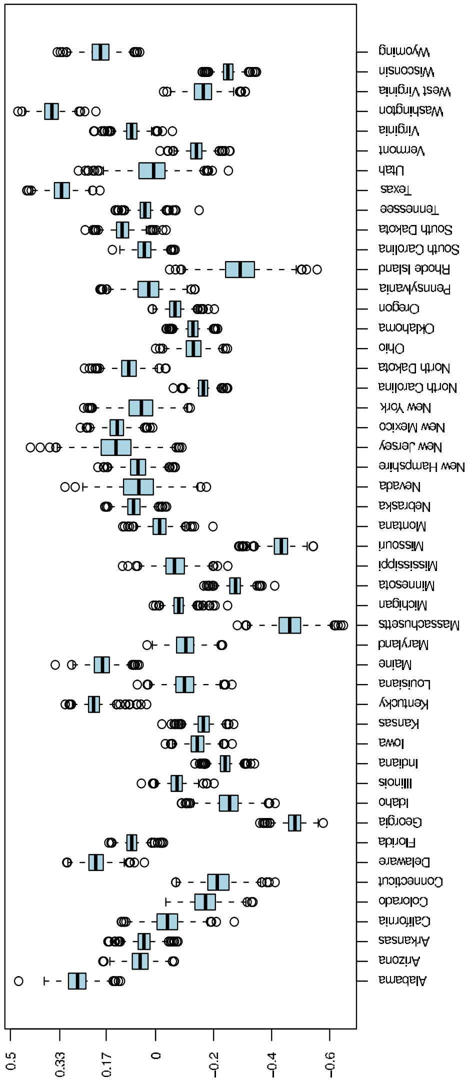

Figure 5.13: Distribution of the Spatial Effects of the Nintendo Wii during First Diffusion Period

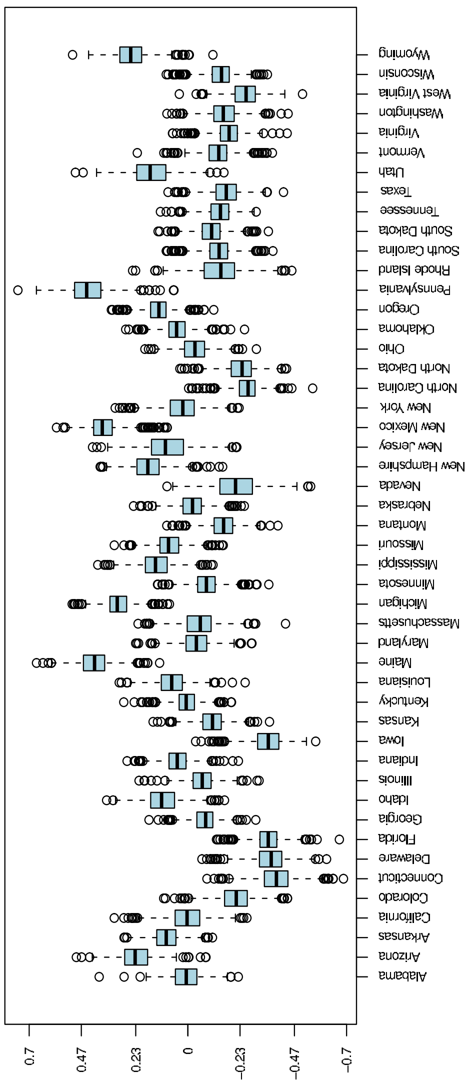

Figure 5.14: Distribution of the Spatial Effects of the Nintendo Wii during Second Diffusion Period

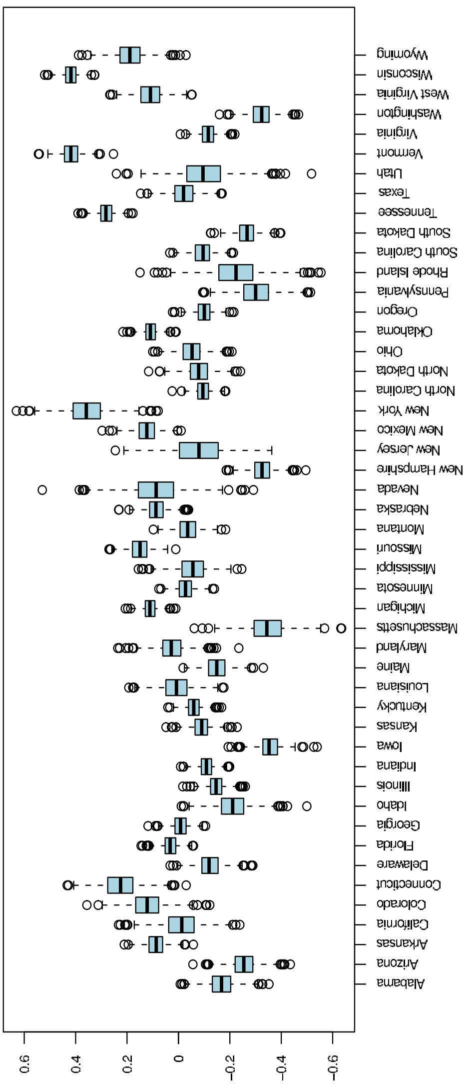

Figure 5.15: Distribution of the Spatial Effects of the Nintendo Wii during Third Diffusion Period

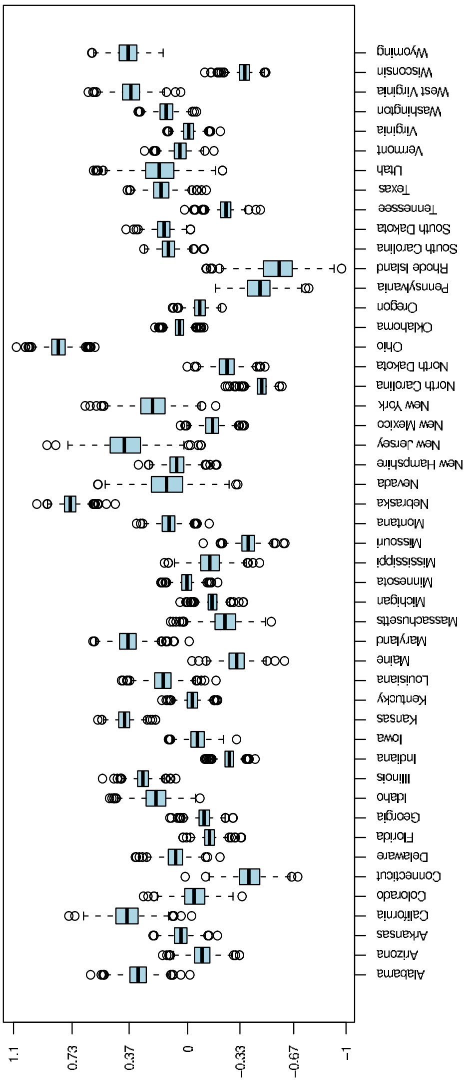

Figure 5.16: Distribution of the Spatial Effects of the Nintendo Wii during Fourth Diffusion Period

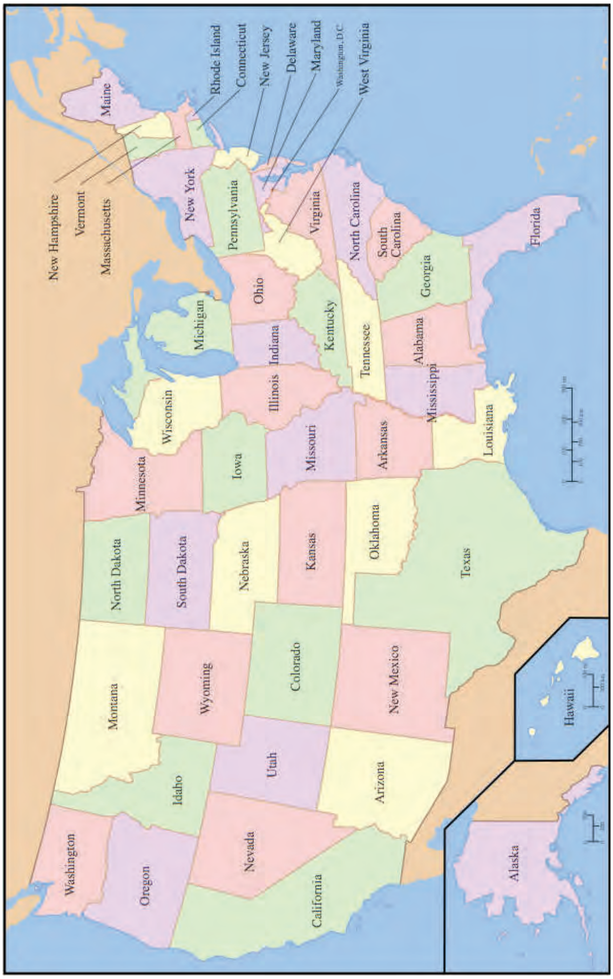

Figure 5.17: US State Map (Source: Wikipedia)

5.A Methodology

In this appendix we discuss the BVS method and the MCAR model estimation we use to study the probabilities of inclusion of the different regions and locations.

Bayesian Variable Selection

In what follows we follow closely the presentation of George and McCulloch (1997) section 4. In Section 4 they discuss the specification of conjugate priors for β and σ. We chose to use conjugate priors because it facilitates the integration of β and σ out of the posterior distribution of the indicators γ and hence the computation of the posterior of γ becomes simple and fast.

The likelihood is specified as

$$f(Y|\beta, \sigma) = \phi(Y; X_\gamma \beta_\gamma, \sigma^2 I) \tag{5.4}$$

where $Y = y_i = (y_{i1}, \ldots, y_{iT})$, X_γ is a subset of potential regressors for which $\gamma = 1$, I is an identity and $\phi(y; x, \Sigma)$ is the Normal distribution density with mean x and variance Σ evaluated at y. The prior for β is

$$\pi(\beta|\sigma, \gamma) = \phi(\beta; 0, \sigma^2 D_\gamma R D_\gamma), \tag{5.5}$$

where D_γ is a diagonal matrix with elements

$$D_\gamma^{kk} = \begin{cases} \upsilon_0 & \text{when} \quad \gamma_k = 0 \\ \upsilon_1 & \text{when} \quad \gamma_k = 1, \end{cases} \tag{5.6}$$

and R is a correlation matrix. $R \propto I$ or $R \propto (X_\gamma' X_\gamma)^{-1}$ are attractive choices when $\upsilon_0 = 0$. The scalars υ_0 and υ_1 are chosen according to the objectives of the modeler. The choice of υ_0 and υ_1 affect the number of regressors included in the subset X_γ and the threshold after which an element of β is distinguished from zero. See George and McCulloch (1997, page 346-347) for more details.

George and McCulloch (1997) discuss how different choices of υ_0 and υ_1 affect the selection of variables and the size of the β coefficients that are included in the model. The suggestion is to set υ_0 small and υ_1 large such that when the posterior supports that $\gamma_k = 0$ then the prior specification is narrow enough to keep β_k close to zero. A popular choice in the literature is to set $\upsilon_0 = 0$ and to specify $\pi(\beta|\gamma) = \pi(\beta_\gamma|\gamma) \times \pi(\beta_{\bar{\gamma}}|\gamma)$ where $\pi(\beta_\gamma|\gamma) = \phi(\beta_\gamma; 0, \sigma^2\Sigma_\gamma)$ and $\pi(\beta_{\bar{\gamma}}|\gamma) = 1$ being β_γ and $\beta_{\bar{\gamma}}$ the coefficients included and excluded in the model, respectively. The attractiveness of this last specification is that we can select β_k depending on how significantly they are different from zero rather than selecting them depending on their relative size when $\upsilon_0 \neq 0$.

The prior for σ^2 is

$$\pi(\sigma^2) = IG(\nu/2, \nu\lambda/2) \tag{5.7}$$

where ν are the degrees of freedom and λ is the scale of the inverse gamma (IG) distribution. What is left to specify is the prior for the indicators γ. They are usually specified as

$$\pi(\gamma) = \prod_k w_k^{\gamma_k}(1 - w_k)^{1-\gamma_k}, \tag{5.8}$$

where w_k is the probability of including the k regressor in the model. A popular choice in the literature is to use $w_k = w$ and therefore

$$\pi(\gamma) = w^{q_\gamma}(1 - w)^{p-q_\gamma}, \tag{5.9}$$

where q_γ is the number of regressors included out of a total set of size p. This last prior can be combined with a conjugate prior on w and set $w \sim Beta(a, b)$ and the prior becomes

$$\pi(\gamma) = \frac{B(a + q_\gamma, b + p - q_\gamma)}{B(a, b)}, \tag{5.10}$$

where $B(x, y)$ is the beta function with x and y parameters. See Chipman et al. (2001) for other choices of $\pi(\gamma)$. Careful selection should be given to the scalars υ_1 and w (or a and b) as they directly affect model size. Large υ_1 and small w concentrate the prior on parsimonious models with large coefficients while large w and small υ_1 concentrate the prior on saturated models with small coefficients (Clyde and George, 2004, page 86).

The joint density $\pi(Y, \beta, \sigma^2|\gamma) = \pi(Y|\beta, \sigma^2, \gamma)\pi(\beta|\sigma, \gamma)\pi(\sigma^2|\gamma)$ has a closed form expression when $\upsilon_0 = 0$ and after integrating over β and σ^2 and that is

$$\pi(Y|\gamma) \propto |X'_\gamma X_\gamma + \Sigma_\gamma^{-1}|^{-1/2}|\Sigma_\gamma|^{-1/2}(\nu\lambda + S^2_\gamma)^{-(T+\nu)/2}, \tag{5.11}$$

where

$$S^2_\gamma = Y'Y - Y'X_\gamma(X'_\gamma X_\gamma + \Sigma_\gamma^{-1})X'_\gamma Y, \tag{5.12}$$

and $\Sigma_\gamma = D_\gamma R D_\gamma$. The posterior of the indicators is straightforward to compute as $\pi(\gamma|Y) \propto \pi(Y|\gamma)\pi(\gamma)$ and the Metropolis-Gibbs sampler is straightforward and it proceeds by sampling $\pi(\gamma|Y)$, $\pi(\beta_\gamma|Y, \sigma^2, \gamma)$ and $\pi(\sigma^2|Y, \beta_\gamma, \gamma)$ sequentially.

We use $a = 50$ and $b = 100$ for the prior on w (in equation (5.10)). The prior of σ^2 has $\nu = 1000$ and $\lambda = 0.30$. We follow the recommendation of George and McCulloch (1997, page 341) who suggest to set λ such that the posterior of σ^2 assigns substantial probability to an interval close to the sample variance of Y and the variance of the residual of a saturated model. The prior on β in equation (5.5) and (5.6) has $\upsilon_0 = 0$ and $\upsilon_1 = 7$ and we use $R = (X'_\gamma X_\gamma)^{-1}$.

A short review of aereal data models

Aereal data usually refers to cross sectional or panel data collected across different regions or areas with well defined boundaries. Therefore aereal data consists of aggregate or summary measures at different locations. The CAR and SAR models are among the most popular models applied to aereal data but there are many other popular approaches like *kriging* or spatial interpolation. In this review we focus on the CAR model and its multivariate extensions.

CAR stands for *Conditional Autoregressors* and SAR stands for *Simultaneous Autoregressors* and hence CAR models are usually referred as Conditionally Autoregressive models and the SAR as Simultaneous Autoregressive models.

The CAR and SAR models are discussed in several sources. A basic reference is Cressie (1992). Cressie covers topics that range from model specification, classical and Bayesian estimation to the theoretical foundations of the CAR and SAR models. Many other topics in spatial analysis are discussed in Cressie (1992). Banerjee et al. (2004) focus on Bayesian analysis and estimation of spatial models. Held and Rue (2002) review many of the computational methods and sampling techniques usually applied to the Bayesian analysis of CAR models and to more general spatial models referred to as *Gaussian Markov Random Fields*.

Wall (2004) compares the CAR and SAR models and offers some insights about the different correlation between locations implied by these two models. The CAR and SAR models might be equivalent under certain conditions, for example see Assunçao (2003) or Banerjee et al. (2004, page 86). We intend to apply spatial priors to the distribution of model parameters. Therefore, in what follows we focus on the CAR model as it is better suited than the SAR both as a hierarchical prior specification on a model's parameters and for Bayesian modeling (Banerjee et al., 2004, page 86).

The main assumption of the CAR model is that a measurement at a location has a conditional distribution with a mean that is proportional to a weighted sum of the measurement at neighboring locations. Both the joint distribution and the conditional distribution of the spatial effects given all other spatial parameters can be derived in closed form and they are presented in Banerjee et al. (2004, page 79) and in the references therein. However, there are alternative specifications to the joint distribution of the spatial effects and a common approach is to use the pairwise difference specification (Besag et al., 1991). Haran et al. (2003) present how to use block updating when some of the coefficients in a linear regression follow the pairwise difference prior.

The CAR is suited for univariate aereal data and Mardia (1988) presents an extension to the multivariate case, usually referred to as *multivariate CAR* or simply as MCAR. It is common to have more than one measurement at each location and the MCAR allows to model both the correlation among measurements of neighboring sites and the correlation among the different measures across sites. Gelfand and Vounatsou (2003) and Carlin and Banerjee (2003) apply Bayesian analysis to the MCAR of Mardia (1988) and present applications with two and up to five dimensional data. On the other hand, Gamerman et al. (2003) present a multivariate version of the pairwise difference specification (used as a prior) and its sampling schemes.

Other extensions of the CAR model incorporate dynamics into its spatial coefficients. Waller et al. (1997), Nobre et al. (2005) and Gelfand et al. (2005) propose models that use a random walk specification for the mean or for the variance of the spatial effects. Gelfand et al. (2005) provide a review of spatio-temporal models.

Linear Model with CAR Prior

Next we work out the specification and sampling for the model

$$y_i = x_i\beta + \phi_i + \epsilon_i, \tag{5.13}$$

where y_i is measured at i locations for $i = 1, \ldots, p$, x_i is a set of k covariates at i and β is a coefficient column vector $k \times 1$ while ϵ_i and ϕ_i are random effects meant to capture overall variability and spatial heterogeneity, respectively. We define $y^{'} = (y_1, \ldots, y_p)$, $\phi^{'} = (\phi_1, \ldots, \phi_p)$ and $X = (x_1, \ldots, x_k)$. The distribution of ϵ_i is

$$\epsilon \sim N(0, \Sigma), \tag{5.14}$$

where $\epsilon^{'} = (\epsilon_1, \ldots, \epsilon_p)$, $\Sigma = \sigma^2 I$ and σ^2 is the variance of ϵ. $N(\mu, \Sigma)$ refers to a normal distribution with mean μ and covariance matrix Σ. We define $\lambda_\epsilon = 1/\sigma^2$. The prior distribution of the spatial effects ϕ_i follows

$$\phi_i|\phi_{j\sim i} \sim N\big(\sum_{j\sim i} c_{ij}\phi_j, \tau_i^2\big). \tag{5.15}$$

This form states that the distribution of ϕ_i given its j neighbors, denoted as $j \sim i$, has a normal distribution with a mean that is a weighted sum (using weights c_{ij}) of the neighboring values and variance τ_i^2. Besag (1974) shows that the joint distribution of the spatial effects in (5.15) can we written in the form

$$\phi \sim N(0, \Omega), \tag{5.16}$$

where $\phi = (\phi_1, \ldots, \phi_p)$ and Ω is a $p \times p$ symmetric and positive semi-definite or positive definite matrix. In the literature it is common to define the elements of Ω^{-1} directly instead of specifying Ω. For example, Banerjee et al. (2004, page 79) assume that $\tau_i^2 = \tau^2/w_{i+}$ and that $c_{ij} = w_{ij}/w_{i+}$ where w_{ij} takes the value of 1 if $j \sim i$ and zero otherwise and where w_{i+} is the total number of neighbors of i. Given these assumptions $\Omega^{-1} = T^{-1}(I - C)$ and given that T is a diagonal matrix with elements $T_{ii} = \tau^2/w_{i+}$ and $C_{ij} = c_{ij}$ then Ω^{-1} can be written as

$$\Omega^{-1} = \frac{1}{\tau^2}(I_{w_{i+}} - W), \tag{5.17}$$

where $I_{w_{i+}}$ is a diagonal matrix with elements w_{i+} and $W_{ij} = w_{ij}$. This last specification for Ω results in an improper distribution given that the rows of $(I_{w_{i+}} - W)$ sum to zero. A solution to this issue is to specify Ω as

$$\Omega^{-1} = \frac{1}{\tau^2}(I_{w_{i+}} - \rho W), \tag{5.18}$$

where ρ takes a value (between 0 and 1) that makes Ω^{-1} positive definite and consequently the distribution of ϕ becomes proper. For a discussion on the *impropriety* of the CAR distribution and the role of the ρ parameter see Banerjee et al. (2004, page 163), Eberly and Carlin (2000), Sahu and Gelfand (1999) or Best et al. (1999). This latter form implies that

$$\phi_i | \phi_{j \sim i} \sim N\big(\rho \sum_{j \sim i} c_{ij}\phi_j, \tau_i^2\big). \tag{5.19}$$

The distribution of ϕ is usually referred as $CAR(\tau^2)$ when the conditional distributions of the spatial effects are defined as in equation (5.15) and it is referred as $CAR(\rho, \tau^2)$ when its conditional distribution follows (5.19). In what follows we use $\Omega^{-1} = \lambda_\phi Q$ with $Q = I_{w_{i+}} - \rho W$ and $\lambda_\phi = 1/\tau^2$. To carry out Bayesian inference and to complete the model specification we need to define the priors for β, λ_y, λ_ϕ and ρ. We specify them as

$$\begin{aligned} p(\beta) &\propto 1 \\ p(\lambda_y) &\propto {\lambda_y}^{a_y} e^{-b_y \lambda_y} \\ p(\lambda_\phi) &\propto \lambda_\phi^{a_\phi} e^{-b_\phi \lambda_\phi} \\ p(\rho) &\propto \text{discretized prior} \end{aligned} \tag{5.20}$$

We use $p(\cdot)$ generically to denote a probability density. That is, the prior for β is non-informative, the priors for λ_y and λ_ϕ have the form of a Gamma distribution. Finally, for ρ we give probability mass to a discrete set of values with a high proportion of them near 1. Gelfand

and Vounatsou (2003) suggest the use of discretized priors for ρ. The model specification is now complete and next we describe the sampling steps to estimate equation (5.13).

Sampling Steps for the CAR

To sample the parameters of the model in equation (5.13) we can apply the Gibbs sampler and MCMC. To derive the posterior of β we can write the likelihood of equation (5.13) as

$$L(y|\beta, \lambda_y) \propto |M|^{-1/2} e^{-\frac{1}{2}(y-X\beta)' M^{-1}(y-X\beta)}, \tag{5.21}$$

where $M = (\frac{1}{\lambda_\phi} Q^{-1} + \frac{1}{\lambda_\epsilon} I)$. The posterior of β is then

$$p(\beta|y, \lambda_y, \lambda_\phi) \propto |M|^{-1/2} e^{-\frac{1}{2}(\beta-b)^{'}(X'M^{-1}X)^{-1}(\beta-b)}, \tag{5.22}$$

with $b = (X'M^{-1}X)^{-1}X'M^{-1}y$. Therefore β can be sampled from $N(b, (X'M^{-1}X)^{-1})$.

Next we derive the posterior distribution of the spatial effects ϕ. To do so we write the density of y conditional on β. That is

$$L(y|\beta, \phi, \lambda_y) \propto \lambda_y^{p/2} e^{-\frac{\lambda_y}{2}(\tilde{y}-\phi)^{'}(\tilde{y}-\phi)}, \tag{5.23}$$

with $\tilde{y} = y - X\beta$. Therefore, the posterior of ϕ is

$$p(\phi|\tilde{y}, \lambda_y, \lambda_\phi) \propto \lambda_y^{p/2} e^{-\frac{1}{2}((\phi-a)^{'} R^{-1}(\phi-a))}, \tag{5.24}$$

where $a = (\lambda_y I + \lambda_\phi Q)^{-1}\lambda_y \tilde{y}$ and $R^{-1} = (\lambda_y I + \lambda_\phi Q)$. That is ϕ can be sampled form $N(a, R)$.

The posterior of λ_y and λ_ϕ are

$$\begin{aligned} p(\lambda_\phi|\tilde{y}, \phi, \lambda_y) &\propto \lambda_\phi^{p/2+a_\phi} e^{-\lambda_\phi(\frac{1}{2}\phi^{'}Q\phi+b_\phi)} \\ p(\lambda_y|\tilde{y}, \phi, \lambda_\phi) &\propto \lambda_y^{p/2+a_y} e^{-\lambda_y(\frac{1}{2}(\tilde{y}-\phi)^{'}(\tilde{y}-\phi)+b_y)}. \end{aligned} \tag{5.25}$$

That is $\lambda_\phi \sim \Gamma(p/2 + a_y, b_y + 1/2\phi^{'}Q\phi)$ and $\lambda_y \sim \Gamma(p/2 + a_\phi, b_\phi + 1/2(\tilde{y} - \phi)^{'}(\tilde{y} - \phi))$.

Finally we need to sample the ρ in the Q matrix. We know that

$$p(\rho|\phi, y, \lambda_y, \lambda_\phi) \propto |Q|^{1/2} e^{-\frac{1}{2}\phi' Q\phi} p(\rho). \tag{5.26}$$

A common method to sample ρ is to assume that $p(\rho)$ is a uniform distribution with range $(0, 1)$ and to sample it with the Metropolis-Hastings algorithm. A second popular choice is to discretize ρ in a set of values and to draw them proportional to their posterior probability. We use the following set $0.01, 0.10, 0.20, 0.30, \ldots, 0.70, 0.71, 0.72, \ldots, 0.99$.

In summary we use the next steps in the Gibbs sampler

1. $\beta \sim N((X'M^{-1}X)^{-1}X'M^{-1}y, (X'M^{-1}X)^{-1})$
2. $\phi \sim N((\lambda_y I + \lambda_\phi Q)^{-1}\lambda_y \tilde{y}, (\lambda_y I + \lambda_\phi Q))$
3. $\lambda_y \sim \Gamma(p/2 + a_y, b_y + 1/2\phi' Q\phi)$
4. $\lambda_\phi \sim \Gamma(p/2 + a_\phi, b_\phi + 1/2(\tilde{y} - \phi)'(\tilde{y} - \phi))$
5. $\rho \sim p(\rho|\phi, y, \lambda_y, \lambda_\phi)$

where $x \sim \Gamma(a, b)$ means that x follows a Gamma distribution with the form $cx^a e^{-bx}$ where c is a constant. At the end of the sampling step 2 we center the ϕ vector around its own mean following Eberly and Carlin (2000) and Best et al. (1999). The re-centering is equivalent to sampling with the restriction $\sum \phi_i = 0$. Rue and Held (2005) show a general form to sample with linear restrictions and that is equivalent to centering around a mean.

Multivariate Linear Model with MCAR Prior

Next we expand the linear model of Section 5.A to a multivariate setting. The exposition follows Carlin and Banerjee (2003) and Gelfand and Vounatsou (2003).

In this setting we observe J different measurements at each location. That is we use the notation y_{ji} to refer to the j^{th} measurement at location i. We use the notation y_j for $(y_{j1}, \ldots, y_{jp})'$ and Y is a $p \times J$ matrix with columns $(y_1, \ldots, y_J)$. The same notation is used for the spatial effects ϕ_{ij} and the error terms ϵ_{ij}. That is $\phi_j = (\phi_{j1}, \ldots, \phi_{jp})'$, $\Phi = (\phi_1, \ldots, \phi_J)$ and finally

$\epsilon_j = (\epsilon_{j1}, \ldots, \epsilon_{jp})'$, $E = (\epsilon_1, \ldots, \epsilon_J)$. We observe a common group of N covariates X where $X = (x_1, \ldots, x_N)$ and $x_i = (x_{i1}, \ldots, x_{ip})'$. Hence we can write

$$\underset{(p\times J)}{Y} = \underset{(p\times N)}{X} \cdot \underset{(N\times J)}{B} + \underset{p\times J}{\Phi} + \underset{(p\times J)}{E} \tag{5.27}$$

To carry out Bayesian inference we define the following priors

$$\begin{aligned} &p(B) \propto 1 \\ &p(\Sigma) \propto |\Sigma|^{-\frac{v}{2}} e^{-\frac{1}{2} tr \Sigma^{-1} V_\Sigma} \\ &p(\Phi|\Lambda, \Psi) \propto |\Psi|^{-J/2} |\Lambda|^{-p/2} e^{-\frac{1}{2} tr(\Psi\Phi\Lambda\Phi')} \\ &p(\Lambda) \propto |\Lambda|^{-\frac{v_0}{2}} e^{-\frac{1}{2} tr \Lambda V_\Lambda} \end{aligned} \tag{5.28}$$

Above Σ is a $J \times J$ covariance matrix of E and $\vec{(E)} \sim N(0, \Sigma \otimes I)$; Λ is $J \times J$ and it is the inverse of the covariance matrix between the columns of Φ while Ψ is $p \times p$ and it is the inverse covariance matrix between the rows of Φ. That is, $\vec{(\Phi)} \sim N(0, \Lambda^{-1} \otimes \Psi^{-1})$.

The form of Ψ might be identical to the form of the Q matrix in the CAR prior. That is $\Psi = (I_{w_{i+}} - \rho W)$ where W and $I_{w_{i+}}$ are defined as before. A second choice for Ψ might be $\Psi = (I_{w_{i+}} - W)$. Carlin and Banerjee (2003) and Gelfand and Vounatsou (2003) use the first form while Gamerman et al. (2003) use the second. A third choice is to define a general form for $\Lambda \otimes \Psi$ as Gelfand and Vounatsou (2003) propose. Gelfand and Vounatsou (2003) propose a form of Q that allows an item (J items) specific ρ parameters. They first define $Q_j = (I_{w_{i+}} - \rho_j W)$ and its Choleski factorization $Q_j = P_j' P_j$. Then they define

$$\Lambda \otimes \Psi = \mathbf{P}'(\Lambda \otimes I_{p\times p})\mathbf{P}, \tag{5.29}$$

where $\mathbf{P}$ is a diagonal matrix with P_j blocks. This last form may allow for a more flexible correlation structure of the Φ parameters. In the application we assume $\rho_j = \rho$ for all j.

Sampling the Multivariate Linear Model with MCAR Prior

If we condition on Φ and define $\bar{Y} = Y - \Phi$ we obtain the traditional multivariate regression model

$$\bar{Y} = X \cdot B + E. \tag{5.30}$$

Given this last expression we can write the density of the model as

$$p(\bar{Y}|X, B, \Sigma) \propto |\Sigma|^{-p/2} e^{-\frac{1}{2}tr(\bar{Y}-XB)'(\bar{Y}-XB)\Sigma^{-1}}. \tag{5.31}$$

The joint posterior of B and Σ can be written as

$$\begin{aligned} p(B, \Sigma|X, Y) &= p(Y|X, B, \Sigma)p(B)p(\Sigma) \\ &\propto |\Sigma|^{-\frac{p+v}{2}} e^{-\frac{1}{2}tr\Sigma^{-1}G}, \end{aligned} \tag{5.32}$$

where $G = (\bar{Y} - XB)'(\bar{Y} - XB) + V_\Sigma$. Furthermore, we can write $G = S + V + (B - \tilde{B})'(X'X)(B - \tilde{B})$ where $S = (\bar{Y} - X\tilde{B})'(\bar{Y} - X\tilde{B})$ and $\tilde{B} = (X'X)^{-1}X'\bar{Y}$. This last form of G allows us to easily integrate out either B or Σ in the last equation and to obtain the posteriors of B and Σ respectively. Therefore

$$\begin{aligned} p(B|X, Y, \Sigma) &\propto |\Sigma|^{-\frac{p+v}{2}} e^{\Sigma^{-1}(B-\tilde{B})'(X'X)(B-\tilde{B})} \\ p(\Sigma|X, Y) &\propto |\Sigma|^{-\frac{p+v}{2}} e^{-\frac{1}{2}tr\Sigma^{-1}(V_\Sigma+S)}, \end{aligned} \tag{5.33}$$

and we can sample B and Σ using these last forms for a matric-variate normal for B and a Inverse Wishart for Σ.

If we condition equation (5.27) on B and we take $\tilde{Y} = Y - XB$ then we have a multivariate regression model

$$\underset{(p\times J)}{\tilde{Y}} = \underset{(p\times J)}{\Phi} + \underset{(p\times J)}{E}, \tag{5.34}$$

and given equation (5.34) we can write the density of $\tilde{Y}$ as

$$p(\tilde{Y}|\Phi, \Sigma) \propto |\Sigma|^{-p/2} e^{-\frac{1}{2}tr(\tilde{Y}-\Phi)'(\tilde{Y}-\Phi)\Sigma^{-1}}. \tag{5.35}$$

If we use $\phi = \vec{(}\Phi)$, $y = \vec{(}\tilde{Y})$ then equation (5.35) can be expressed as

$$p(y|\phi, \Sigma) \propto |\Sigma|^{-p/2} e^{-\frac{1}{2}(y-\phi)^{'}(\Sigma^{-1} \otimes I_{p \times p})(y-\phi)}. \tag{5.36}$$

In the same way the prior for Φ can be expressed in vectorized form as

$$p(\phi) \propto |\Psi|^{-J/2} |\Lambda|^{-p/2} e^{-\frac{1}{2}\phi'(\Psi \otimes \Lambda)\phi}. \tag{5.37}$$

We use the vectorized forms to derive the posterior of ϕ. That is $p(\phi|y, \Sigma, \Psi) \propto p(y|\phi, \Sigma) \times p(\phi)$ and therefore

$$p(\phi|y, \Sigma, \Psi) \propto |\Lambda|^{-\frac{(2p+v_0)}{2}} |\Sigma|^{-p/2} e^{-\frac{1}{2}((\phi-a)'M^{-1}(\phi-a)+S_\phi)} \tag{5.38}$$

where $S_\phi = y'Hy + a'M^{-1}a$, $M^{-1} = (H + F)$, $H = \Sigma^{-1} \times I$, $F = \Psi \otimes \Lambda$ and $a = MHy$.

The posterior of Λ can be derived from the third and fourth line of equation (5.28) as follows

$$p(\Lambda|\Phi, Y, \Sigma, \Psi) \propto |\Lambda|^{-\frac{(p+v_0)}{2}} e^{-\frac{1}{2}tr\Lambda(V_\Lambda + \Phi^{'}\Psi\Phi)}. \tag{5.39}$$

If the form of Ψ contains a ρ or ρ_j parameters Gelfand and Vounatsou (2003) suggest to sample them from a discretized prior. The posterior of the ρ parameters is

$$p(\rho|\Phi, Y, \Sigma, \Lambda) \propto |\Psi|^{-J/2} e^{-\frac{1}{2}tr(\Psi\Phi^{'}\Lambda\Phi)}. \tag{5.40}$$

In summary we use the following Gibbs steps

1. $\beta|X, \bar{Y}, \Phi, \Lambda, \Psi \sim N(\vec{(}(X'X)^{-1}X'\bar{Y}), \Sigma \otimes (X'X)^{-1})$

2. $\phi|B, X, Y, \Lambda, \Psi \sim N((\Sigma^{-1} \otimes I + \Psi \otimes \Lambda)^{-1}(\Sigma^{-1} \otimes I)y, (\Sigma^{-1} \otimes I + \Psi \otimes \Lambda))$

3. $\Sigma|Y, B, \Phi, \Lambda, \Psi \sim IW((p + v)/2, V_\Sigma + S)$

4. $\Lambda|\Psi, B, X, Y, \Sigma \sim IW((p + v_0)/2, V_\Lambda + \Phi^{'}\Psi\Phi)$

5. $\rho|\Phi, \Lambda, B, X, Y, \Sigma \sim p(\rho|\Phi, Y, \Sigma, \Lambda)$

In the chapter we set $V_\Sigma = I_3$ and $V_\Lambda = I_3$ and $\upsilon_0 = 5$ while $\upsilon = 3$ and $p = 48$. We use 48 states because we leave out Hawaii and Alaska. The matrix Ψ is defined based on the neighborhood structure of the US states where the element Ψ_{kj} takes the value of one when the state k is neighbor of the state j and zero otherwise. We further assume that $\rho_j = \rho$ and we sample this parameter based on the discretized prior described above. Finally, we assume that $\Sigma = \sigma^2 I$ and the β coefficients are equal across technologies.

Nederlandse Samenvatting (Summary in Dutch)

Dit proefschrift richt zich op de analyse van nieuwe of zeer recente marketing gegevens. Hiertoe introduceren we een aantal nieuwe econometrische modellen. We presenteren modellen die nuttig zijn om het volgende te analyseren: (1) het optimale tijdstip voor de lancering van nieuwe en dominante technologieën, (2) de *triggers*, snelheid en de timing van een substantiële prijsverlaging voor nieuwe producten, (3) de heterogeniteit in preferenties van consumenten die leidt tot specifieke substitutiepatronen in geaggregeerde verkoopgegevens, en (4) locaties die een grote invloed hebben op de verspreiding van nieuwe technologieën. De econometrische technieken die we toepassen zijn divers, maar ze zijn voornamelijk gebaseerd op Bayesiaanse methoden. We maken gebruik van Bayesiaanse *mixture* modellen, Bayesiaanse variabele selectie technieken, Bayesiaanse *spatial* modellen en we introduceren een nieuwe Bayesiaanse benadering voor het random coëfficiënten logit model. De gegevens die we analyseren bestaan uit unieke en grote datasets. We bestuderen de prijzen van video-games, de verkopen van video-game consoles, de totale omzet voor specifieke consumentenproducten en online zoekgegevens van Google.

Resumen en Español (Summary in Spanish)

En esta tesis se analizan nuevas bases de datos de mercadotecnia y se presentan nuevos modelos econometricos. Estos nuevos modelos son útiles para analizar (1) el tiempo de lanzamiento óptimo de nuevas tecnologías, (2) los factores que provocan cortes drásticos en los precios de nuevos productos y a la vez la velocidad y el momento en el que ocurren los cortes, (3) la heterogeneidad de los consumidores que determina los patrones de sustitución presentes en datos de ventas agregados, y (4) los mercados influyentes que determinan la difusión de nuevas tecnologías. Los métodos econométricos que se utilizan en esta tesis son diversos pero en su mayoría son métodos Bayesianos. Usamos modelos de mezcla de distribuciones, técnicas Bayesianas de selección de variables, modelos Bayesianos para datos geográficos y proponemos un nuevo enfoque Bayesiano para el modelo logit con coeficientes aleatorios. Los datos que se analizan son precios de videojuegos, ventas de consolas de videojuegos, datos agregados de ventas de productos de consumo y datos de búsqueda *en línea* de Google.

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Author Index

About the Author

Carlos Hernández Mireles received a B.A. in Economics with honors from ITESM in 2001 and he obtained the M.Phil. degree in Marketing from the Erasmus Research Institute of Management (ERIM) in 2006. In November of 2006, he started his Ph.D. research in ERIM and the Econometric Institute. His main research interests are marketing modeling and Bayesian methods. His research has been presented at several international conferences. Prior and during his Ph.D. Carlos worked for Cemex, Unilever and Heineken.

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MARKETING MODELING FOR NEW PRODUCTS

This thesis addresses the analysis of new or very recent marketing data and the introduction of new marketing models. We present a collection of models that are useful to analyze (1) the optimal launch time of new and dominant technologies, (2) the triggers, speed and timing of new products' price landings, (3) the consumer heterogeneity that drives substitution patterns present in aggregate data, and (4) the influential locations that drive the diffusion of new technologies. The econometric approaches that we apply are diverse but they are predominantly Bayesian methods. We use Bayesian mixture modelling, Bayesian variable selection techniques, Bayesian spatial models and we put forward a new Bayesian approach for the random coefficient logit model. The data that we analyze consist of unique and large datasets of video-game prices, video-game consoles' sales, aggregate sales data for consumer products and Google's online search data.

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